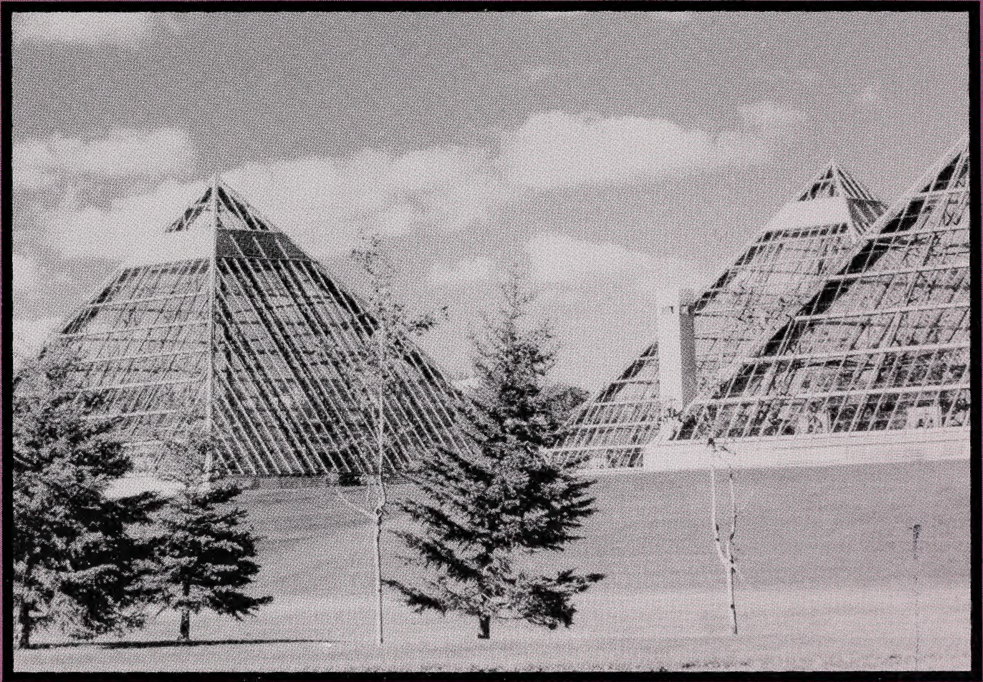
MATHEMATICS 10

UNIT 7



TRIGONOMETRY

Distance Learning

Alberta EDUCATION

Welcome

Distance Learning

You have chosen an alternate form of learning that allows you to work at your own pace. You will be responsible for your own schedule, for disciplining yourself to study the units thoroughly, and for completing your units regularly. We wish you much success and enjoyment in your studies.

Mathematics 10 Student Module Unit 7 Trigonometry Alberta Distance Learning Centre ISBN No. 0-7741-0753-7

This document is intended for	
Students	✓
Teachers (Mathematics 10)	✓
Administrators	
Parents	
General Public	
Other	

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General Information

This information explains the basic layout of each booklet.

- **What You Already Know** and **Review** are to help you look back at what you have previously studied. The questions are to jog your memory and to prepare you for the learning that is going to happen in this unit.

- As you begin each **Topic**, spend a little time looking over the components. Doing this will give you a preview of what will be covered in the topic and will set your mind in the direction of learning.

- **Exploring the Topic** includes the objectives, concept development, and activities for each objective. Use your own papers to arrive at the answers in the activities.

- **Extra Help** reviews the topic. If you had any difficulty with **Exploring the Topic**, you may find this part helpful.

- **Extensions** gives you the opportunity to take the topic one step further.

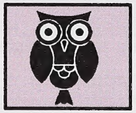
- To summarize what you have learned, and to find instructions on doing the unit assignment, turn to the **Unit Summary** at the end of the unit.

- The **Appendices** include the solutions to **Activities (Appendix A)** and any other charts, tables, etc. which may be referred to in the topics **(Appendix B, etc.)**.

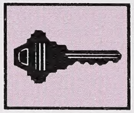
Visual Cues

Visual cues are pictures that are used to identify important areas of the material. They are found throughout the booklet.

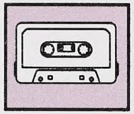
An explanation of what they mean is written beside each visual cue.



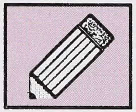
What You Already Know
- reviewing what you already know



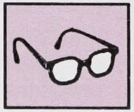
Key Idea
- flagging important ideas



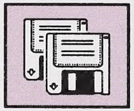
Audiotape
- learning by listening to an audiotape



Review
- studying previous concepts

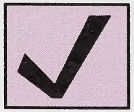


Another View
- exploring different perspectives

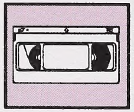


Computer Software
- learning by using computer software

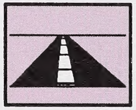
Introduction
- introducing the unit



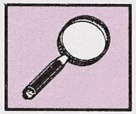
Solutions
- correcting the activities



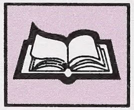
Videotape
- learning by viewing a videotape



What Lies Ahead
- previewing the unit

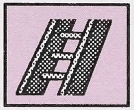


Extra Help
- providing additional study

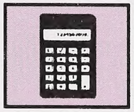


Print Pathway
- choosing a print alternative

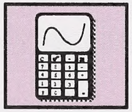
Exploring the Topic
- actively learning new concepts



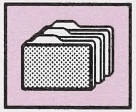
Extensions
- going on with the topic



Calculator
- using your calculator



Graphing Calculator
- using your graphing calculator



What You Have Learned
- summarizing what you have learned

Mathematics 10

Course Overview

Mathematics 10 contains 8 units. Beside each unit is a percentage that indicates what the unit is worth in relation to the rest of the course. The units and their percentages are listed below. You will be studying the unit that is shaded.

Unit 1 Number Systems	10%
Unit 2 Operations on Polynomials	14%
Unit 3 Equations and Inequalities	10%
Unit 4 Factoring Polynomials	13%
Unit 5 Coordinate Geometry	19%
Unit 6 Systems of Equations	10%
Unit 7 Trigonometry	11%
Unit 8 Statistics	13%
	100%

Unit Assessment

After completing the unit, you will be given a mark based totally on a unit assignment. This assignment will be found in the Assignment Booklet.

Unit Assignment - 100%

If you are working on a CML terminal, your teacher will determine what this assessment will be. It may be

Unit Assignment - 50%
Supervised Unit Test - 50%

Introduction to Trigonometry

This unit covers topics dealing with trigonometry. Each topic contains explanations, examples, and activities to assist you in understanding trigonometry. If you find you are having difficulty with the explanations and the way the material is presented, there is a section called **Extra Help**. If you would like to extend your knowledge of the topic, there is a section called **Extensions**.

You can evaluate your understanding of each topic by working through the activities. Answers are found in the solutions in the **Appendix**. In several cases there is more than one way to do the question.

Unit 7 Trigonometry

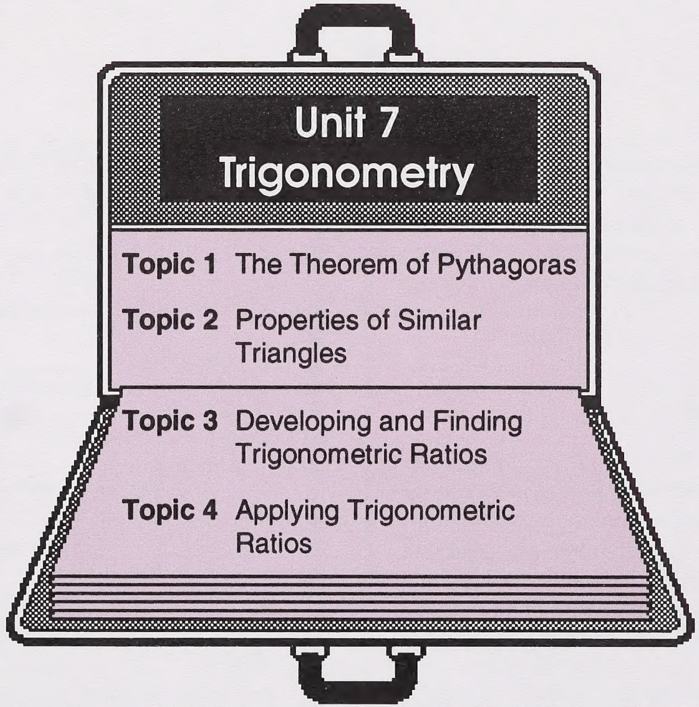
Contents at a Glance

Trigonometry

The study of trigonometry can be traced back to the Greeks, about the third century B.C. Their interest in the movements of the heavenly bodies led them to find ways of calculating and recording the relative positions of the stars they observed. The word trigonometry comes from three Greek words which mean the measurement of three angles. These three words are as follows:

tri	*gono*	*metry*
↓	↓	↓
three	angle	measurement

Putting them together gives you trigonometry. In its earliest development, trigonometry was closely associated with surveying. Today its important applications lie in the fields of electronics, physics, engineering, and advanced chemistry. The calculations in this unit can be done easily with calculators.

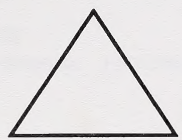

Unit 7
Trigonometry
Topic 1 The Theorem of Pythagoras
Topic 2 Properties of Similar Triangles
Topic 3 Developing and Finding Trigonometric Ratios
Topic 4 Applying Trigonometric Ratios

What You Already Know

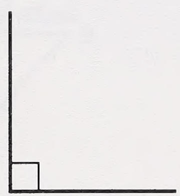
Refresh your memory!

- Review the definitions of some geometric terms.

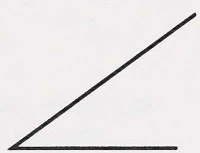
  – A triangle is a three-sided polygon.



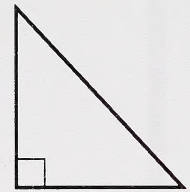
  – A right angle is an angle measuring 90°.



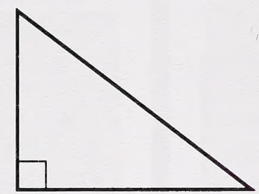
  – An acute angle is an angle measuring less than 90°.



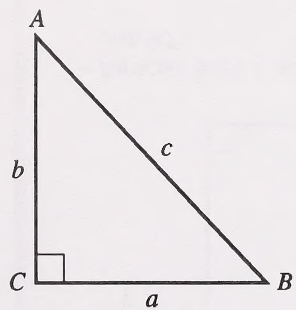
  – A right triangle is a triangle that has one angle equal to 90°.



  – The hypotenuse is the longest side of a right triangle, or the side opposite the right angle in a right triangle.



- Remember the proper labelling of the parts of any triangle. Use capital letters for the vertices and small letters for the sides. Review the example provided. Angle C is usually the right angle.



The lower case letters (a, b, c) refer to the length of the sides ($\overline{CB}$, $\overline{AC}$, and $\overline{AB}$ respectively).

- Remember how to square a number. To square a number, multiply it by itself.

5 squared or 5 to the exponent 2 $\leftarrow 5^2 = 5 \times 5 \rightarrow$ a perfect square

$= 25$

$\downarrow$

5 multiplied by itself

- Recall finding the square root of a perfect square. This is finding a number which when multiplied by itself equals the original perfect square value.

square root or radical sign

$\uparrow$

$\sqrt{36} = \sqrt{6 \times 6}$

$\downarrow$ original perfect square value

$= 6$

$\downarrow$ square root of 36 is 6 because $6 \times 6 = 36$

- Remember how to solve ratios. Ratios can be written in various ways. When you show that one ratio is equal to another, the result is a proportion. To solve ratios or to solve proportions, find the cross products to get a linear equation. Then solve the equation for the variable that is used. The values used in ratios and in proportions do not have to be restricted to whole numbers. Decimals and fractions are other values that are used.

To name the sides you can use line segments. For example,

$\overline{AB}$, $\overline{BC}$, $\overline{AC}$

a = measure of $\overline{CB}$
b = measure of $\overline{AC}$
c = measure of $\overline{AB}$

Note: 3:4, 3 to 4, and $\frac{3}{4}$ are all ratios written in different ways.

Solve for x.

$$\frac{x}{10} = \frac{5}{2}$$

Find the cross products.

$$2 \times x = 10 \times 5$$
$$2x = 50$$

Solve the linear equation for x.

$$2x = 50$$
$$\frac{2x}{2} = \frac{50}{2}$$
$$x = 25$$

The ratios $\frac{25}{10}$ and $\frac{5}{2}$ are equivalent.

Another proportion is shown as follows:

$$\frac{0.75}{6} = \frac{4}{x}$$
$$0.75x = 6 \times 4$$
$$0.75x = 24$$
$$\frac{0.75x}{0.75} = \frac{24}{0.75}$$
$$x = 32$$

The ratios $\frac{0.75}{6}$ and $\frac{4}{32}$ are equivalent.

Now that you have looked at material you studied previously, go to the review to confirm your understanding of this material.

Note: $\frac{0.75}{6}$ can be simplified as follows:

$$\frac{0.75 \times 100}{6 \times 100} = \frac{75}{600}$$
$$= \frac{1}{8}$$

$\frac{4}{32}$ is also equal to $\frac{1}{8}$.

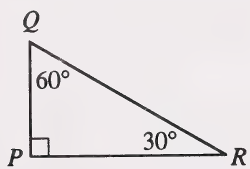
Two ratios are equivalent if the cross products are equal.

$\frac{1}{2} = \frac{9}{18}$ because $1 \times 18 = 2 \times 9$

Therefore, we say $\frac{1}{2}$ and $\frac{9}{18}$ are equivalent.

Review

1. Use the diagram to do the following.



   a. Name the hypotenuse.

   b. Name the right angle.

   c. Name an acute angle.

   d. Name the sides opposite $\angle Q$, opposite $\angle R$, and opposite $\angle P$.

2. Find the square root for each of the following.

   a. 49   b. 81

   c. 225

3. Square each of the following.

   a. 16   b. 25

   c. 64

4. Solve for the unknown in each of the following proportions. Do not expect each one to work evenly.

   a. $\frac{5}{6}=\frac{x}{18}$ b. $\frac{x}{12}=\frac{2}{3}$

   c. $\frac{5}{x}=\frac{15}{27}$ d. $\frac{14}{15}=\frac{56}{x}$

   e. $\frac{0.5}{6}=\frac{x}{0.2}$ (Round to the nearest thousandth.)

   f. $\frac{\frac{5}{6}}{3}=\frac{12}{x}$ g. $\frac{7}{10}=\frac{x}{3\frac{3}{4}}$

   h. $\frac{3}{4.5}=\frac{14}{x}$ i. $\frac{x}{1\frac{3}{4}}=\frac{20}{1.5}$

Now go to the **Review** solutions in the **Appendix**.

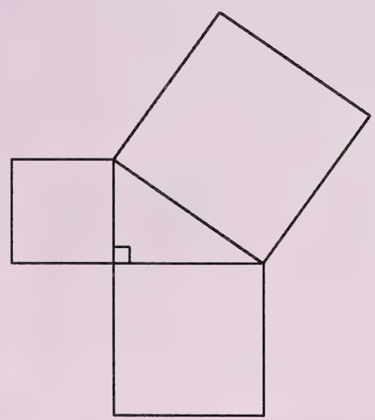
If you had difficulties with the exercises, go to the following modules in **Practical Mathematics**.

- **Module 2, Fractional Numbers**
- **Module 3, Decimal Numbers**
- **Module 4, Ratios and Proportion**

Topic 1 The Theorem of Pythagoras

Introduction

In ancient days (as well as in modern times) groups of clever people often got together to share new ideas and useful discoveries. One such ancient person was a Greek philosopher named Pythagoras. He is remembered historically for many ideas, but he is best known for what we call the Pythagorean theorem. This useful mathematical tool is also referred to as the theorem of Pythagoras.



What Lies Ahead

Throughout this topic you will learn to

1. draw and label the sides and angles of a triangle
2. find the length of a side of a given triangle, and solve problems by applying the Pythagorean theorem.

Now that you know what to expect, turn the page to begin your study of the theorem of Pythagoras.

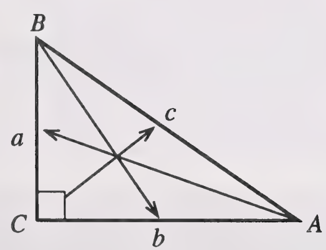
Exploring Topic 1

Activity 1

Draw and label the sides and angles of a triangle.

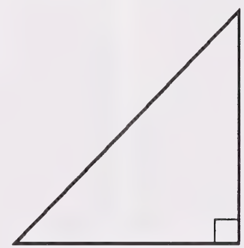
Right-angled triangles are commonly used not only in mathematical problems but also for the solving of many practical situations. Before solving problems, you need to know a standard way of labelling right-angled triangles.

Angles are labelled using capital letters, and sides are labelled using small letters. The same letter is used for both the angles and the side that is opposite that particular angle. Study the following diagram to see the relationship between angles and the opposite sides.

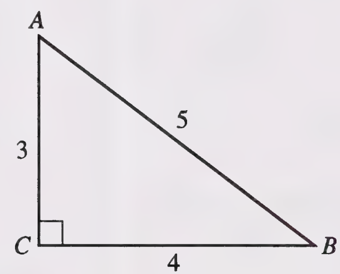


Knowing this pattern, go on to study the Pythagorean theorem in much greater detail. Do the following questions.

1. a. Use C, D, and E to label the vertices in the triangle provided. Correctly label the sides.



   b. Using ΔABC, give the measure of each side.



For solutions to **Activity 1**, turn to the **Appendix**, **Topic 1**.

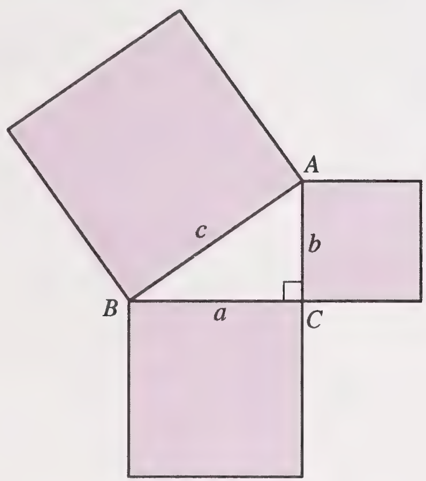
C is generally used to name the right angle.

Note: Side a is opposite angle A; side b is opposite angle B; and side c is opposite angle C.

Activity 2

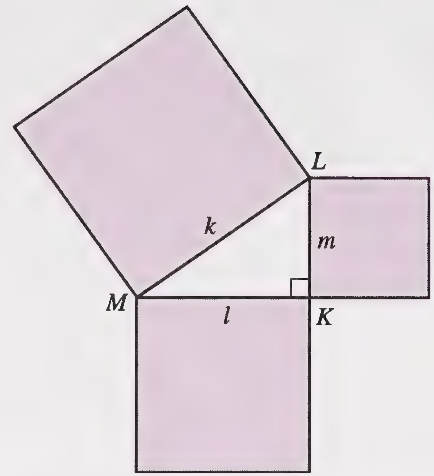
Find the length of a side of a given triangle, and solve problems by applying the Pythagorean theorem.

In any right triangle, the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the other two sides.



For the given triangle, $c^2 = a^2 + b^2$.

The Pythagorean theorem can be expressed using other letters. The letters used depend on how a triangle is labelled. Notice what happens in ΔKLM.



For ΔKLM, $k^2 = l^2 + m^2$.

This situation suggests that a variety of letters can be used in the Pythagorean theorem. The variables used in the Pythagorean theorem depend on the variables used to label the vertices. Usually the Pythagorean theorem is specified as $c^2 = a^2 + b^2$.

The Pythagorean theorem states that if any two sides of a right triangle are known, the third side can be found by using the formula $c^2 = a^2 + b^2$.

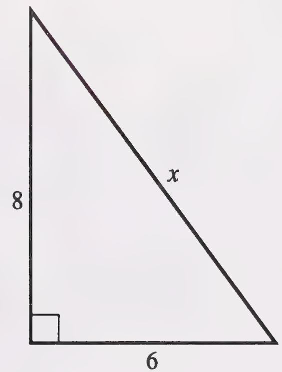
$c^2 = a^2 + b^2$ can be written as $a^2 + b^2 = c^2$, and $k^2 = l^2 + m^2$ can be written as $l^2 + m^2 = k^2$.

Note: The hypotenuse is always opposite the right angle. This is called side c only if the right angle is called $\angle C$.

Note: There are two common adaptions of $c^2 = a^2 + b^2$. These are $a^2 = c^2 - b^2$ and $b^2 = c^2 - a^2$.

Example 1

Find the value of x.



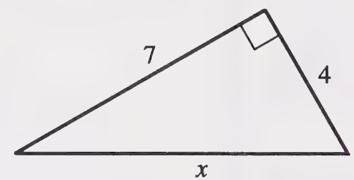
Solution:

x is the hypotenuse.

$$\begin{aligned} x^2 &= 6^2 + 8^2 \\ x^2 &= 36 + 64 \\ x^2 &= 100 \\ \sqrt{x^2} &= \sqrt{100} \\ x &= 10 \end{aligned}$$

Example 2

Find the value of x. Round your answer to one decimal place.



Solution:

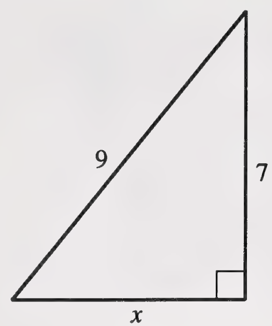
x is the hypotenuse.

$$\begin{aligned} x^2 &= 4^2 + 7^2 \\ x^2 &= 16 + 49 \\ x^2 &= 65 \\ \sqrt{x^2} &= \sqrt{65} \\ x &\doteq 8.062\,257\,748 \\ &\doteq 8.1 \end{aligned}$$

Enter	Display
65	65
$\boxed{\sqrt{\ }}$	8.062257748

Example 3

Find the value of x. Round your answer to one decimal place.



Solution:

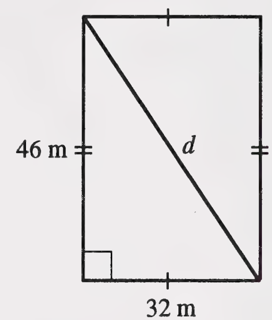
x is not the hypotenuse.

$$\begin{aligned} 9^2 &= x^2 + 7^2 \\ 81 &= x^2 + 49 \\ x^2 &= 81 - 49 \\ x^2 &= 32 \\ \sqrt{x^2} &= \sqrt{32} \quad \text{(Use your calculator.)} \\ x &\doteq 5.656\,854\,249 \\ &\doteq 5.7 \end{aligned}$$

Example 4

The dimensions of a rectangle are 32 m by 46 m. What is the length of the diagonal? Round your answer to two decimal places.

Solution:



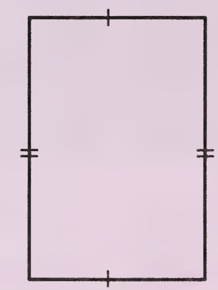
$$\begin{aligned} d^2 &= 32^2 + 46^2 \\ d^2 &= 1024 + 2116 \\ d^2 &= 3140 \\ \sqrt{d^2} &= \sqrt{3140} \\ d &\doteq 56.035\,7029 \\ &\doteq 56.04 \text{ m} \end{aligned}$$

The length of the diagonal is about 56.04 m.

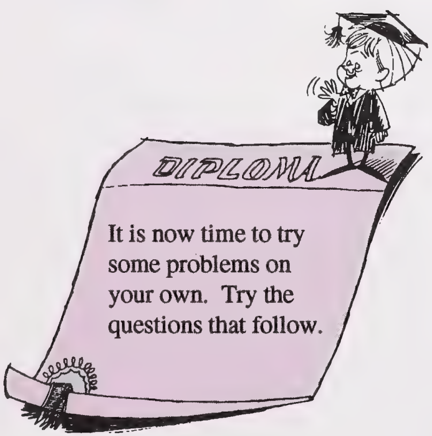
Recall: A diagonal joins two nonconsecutive vertices of a polygon.

The use of a diagram often helps when solving a problem.

The slashes mean that the segments are equal in length. All segments with one slash are equal; all segments with two slashes are equal; and so on.



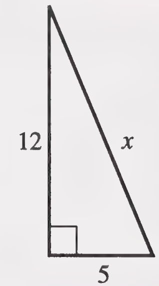
Enter	Display
32	32
$\boxed{x^2}$	1024
$\boxed{+}$	1024
46	46
$\boxed{x^2}$	2116
$\boxed{=}$	3140
$\boxed{\sqrt{\ }}$	56.0357029



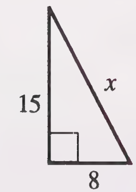
Do any three from questions 1 and 2. Do all of question 3.

1. Solve for x in each of the following.

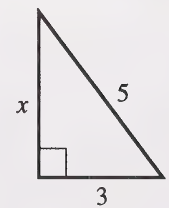
a.



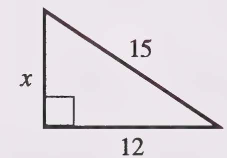
b.



c.



d.



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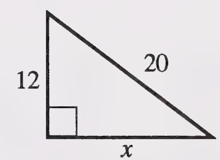

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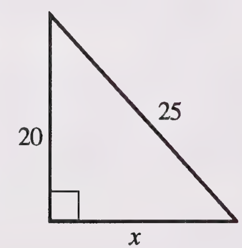
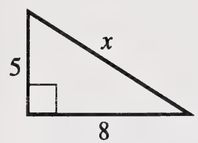


2. Solve for x in each of the following. Round each answer to the nearest tenth.

a.



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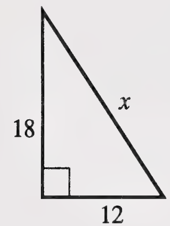
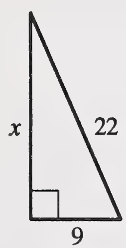


c.



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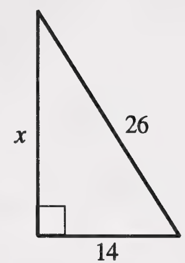

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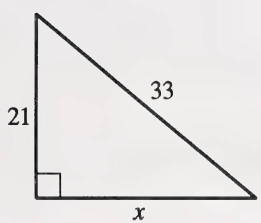

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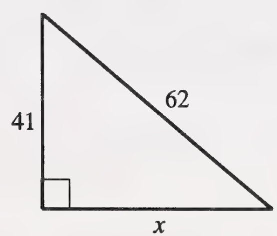


3. A lookout tower spotted some smoke in the southeast. To get to this spot firefighters would have to go across a large lake. To get there by land they would have to go 1200 m south and 1160 m east. What is the direct distance to the fire across the lake? How much shorter is the direct route? Round your answer to the nearest metre. Sketch a diagram to assist you in solving the problem.

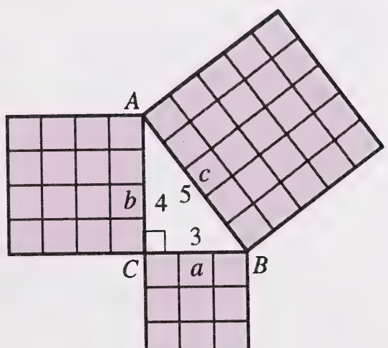
4. Pavel drops one end of a 40 m rope to Susan who is standing at the very bottom of a cliff. When Susan walks away from the cliff to stretch the rope tightly, she finds herself 11 m from the base of the cliff. How high is the cliff? Round your answer to one decimal place if necessary.

For solutions to **Activity 2**, turn to the **Appendix**, **Topic 1**.

If you require help, do the Extra Help section.
If you want more challenging explorations, do the Extensions section.
} You may decide to do both.

Extra Help

The Pythagorean theorem can be illustrated using a simple diagram. Use a right triangle as shown in the following diagram. Label it *ABC*. Take note how the sides are labelled using the small letters for *A*, *B*, and *C*. On side *a* draw a square and divide it into nine smaller squares. On side *b* make a square and divide it into sixteen smaller squares. On side *c*, which is the hypotenuse, draw a square with a total of twenty-five squares. Side *a* is 3 units in length; side *b* is 4 units in length; and side *c* is 5 units in length.



Substitute these values into the Pythagorean theorem.

LS	RS
c^2	$a^2 + b^2$
5^2	$3^2 + 4^2$
25	9+16
25	25
LS =	RS

This example shows that the sum of the squares on the two sides of a right triangle is equal to the squares on the hypotenuse.

As you continue your study of mathematics, you will learn that all cases do not work as nicely and evenly as the numbers 3, 4, and 5.

The number 3, 4, and 5 have a special relationship because $3^2 + 4^2 = 5^2$. These three numbers are often referred to as a Pythagorean triple. There are numerous examples of Pythagorean triples. To see just a few, look at the following partial list. There is no end to listing Pythagorean triples.

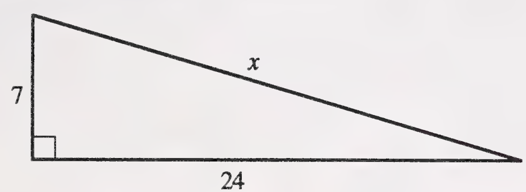
Group 1		Group 2	
3, 4, 5	9, 40, 41	3, 4, 5	24, 32, 40
5, 12, 13	11, 60, 61	6, 8, 10	48, 64, 80
7, 24, 25	13, 84, 85	12, 16, 20	96, 128, 160
	etc.		etc.

For additional practice, do all of the following problems. Use a calculator where possible, and if an exact answer cannot be calculated, round your answer to the nearest tenth (one decimal place).

Do the odd- or even-numbered problems. Do the others if you need more practice.

1. Solve for x in the following triangles.

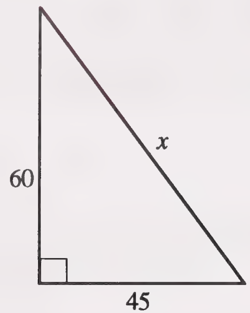
a.



Recall: The hypotenuse is the side opposite the right angle. It is the longest side of any right-angled triangle.

Use a calculator to check if these are Pythagorean triples. Do you see a pattern? Can you come up with others?

b.



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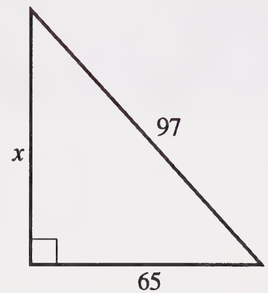
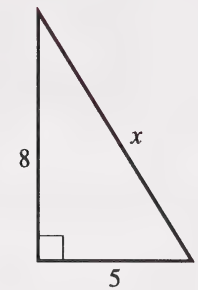


g.



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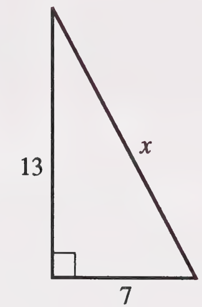

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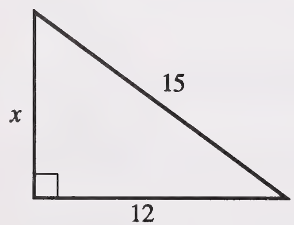

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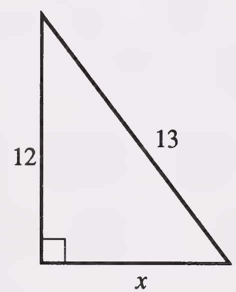
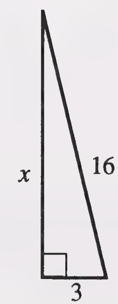
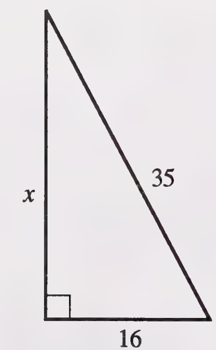


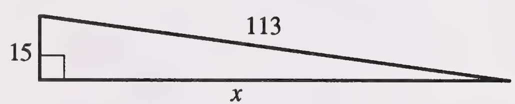
i.



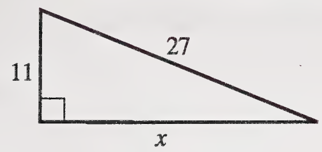
j.



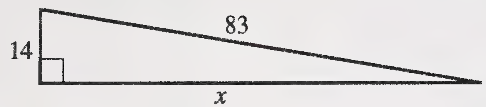
f.



k.



l.



2. The dimensions of a rectangular playing field are 120 m by 75 m. How much closer is it to walk from one corner to the opposite corner rather than along the length and width of the field? Round your answer to the nearest metre.

3. A square gate is 4 m on each side. What is the length of the diagonal brace on a gate of this size? Round you answer to the nearest tenth of a metre.

For solutions to **Extra Help**, turn to the **Appendix**, **Topic 1**.

Extensions

To this point you used whole numbers and decimal numerals to represent the length of each side of a right-angled triangle. In this section you will see that instead of using approximate values, exact values can be used even though exact square roots of certain numbers cannot be found.

These exact values are called radicals. There are two types of radicals that are commonly used. They are as follows:

- Entire Radicals

  $\sqrt{10}$, $\sqrt{3}$, $\sqrt{20}$, $\sqrt{45}$, etc

- Mixed Radicals

  $2\sqrt{3}$, $4\sqrt{5}$, $10\sqrt{2}$, $6\sqrt{11}$, etc

Entire radicals can be changed to mixed radicals, and mixed radicals can be changed to entire radicals.

Example 5

Change each of the following to a mixed radical.

- $\sqrt{20}$

Solution:

$$\begin{aligned}\sqrt{20} &= \sqrt{4\times 5}\\ &= \sqrt{4}\times\sqrt{5}\\ &= 2\sqrt{5}\end{aligned}$$

- $\sqrt{72}$

Solution:

$$\begin{aligned}\sqrt{72} &= \sqrt{36\times 2}\\ &= \sqrt{36}\times\sqrt{2}\\ &= 6\sqrt{2}\end{aligned}$$

- $\sqrt{75}$

Solution:

$$\begin{aligned}\sqrt{75} &= \sqrt{25\times 3}\\ &= \sqrt{25}\times\sqrt{3}\\ &= 5\sqrt{3}\end{aligned}$$

- $\sqrt{700}$

Solution:

$$\begin{aligned}\sqrt{700} &= \sqrt{100\times 7}\\ &= \sqrt{100}\times\sqrt{7}\\ &= 10\sqrt{7}\end{aligned}$$

Example 6

Change each of the following to an entire radical.

- $5\sqrt{5}$

Solution:

$$\begin{aligned}5\sqrt{5} &= \sqrt{25}\times\sqrt{5}\\ &= \sqrt{25\times 5}\\ &= \sqrt{125}\end{aligned}$$

- $4\sqrt{3}$

Solution:

$$\begin{aligned}4\sqrt{3} &= \sqrt{16}\times\sqrt{3}\\ &= \sqrt{16\times 3}\\ &= \sqrt{48}\end{aligned}$$

Recall: $3\sqrt{5}$ is called a mixed radical. The 3 is called a multiplier, and the 5 is called a radicand.

To change an entire radical to a mixed radical, remove the highest possible perfect square factor from the radicand to get the multiplier. The nonperfect square factor remains as the radicand in the mixed radical. The reverse is true when changing a mixed radical to any entire radical.

Note: The multiplier is squared when it is made part of the radicand.

- $20\sqrt{2}$

Solution:

$$\begin{aligned} 20\sqrt{2} &= \sqrt{400}\times\sqrt{2} \\ &= \sqrt{400\times 2} \\ &= \sqrt{800} \end{aligned}$$

- $7\sqrt{10}$

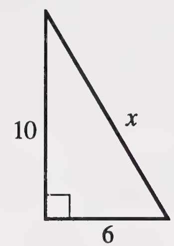
Solution:

$$\begin{aligned} 7\sqrt{10} &= \sqrt{49}\times\sqrt{10} \\ &= \sqrt{49\times 10} \\ &= \sqrt{490} \end{aligned}$$

In the following example, notice how radicals are used when applying the Pythagorean theorem to right-angled triangles.

Example 7

Solve for x in each of the following right triangles. Simplify the radicals where possible.

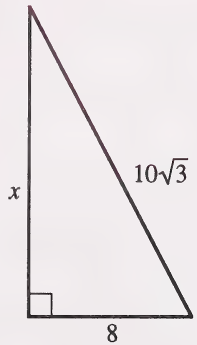
- 

Solution:

$$\begin{aligned} x^2 &= 10^2 + 6^2 \\ x^2 &= 100 + 36 \\ x^2 &= 136 \\ \sqrt{x^2} &= \sqrt{136} \\ x &= \sqrt{4\times 34} \\ &= 2\sqrt{34} \end{aligned}$$

The value of x is $2\sqrt{34}$.

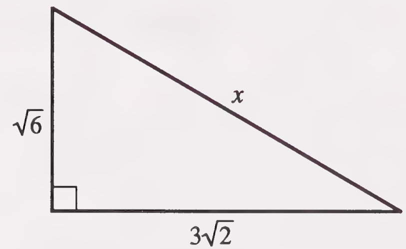
$2\sqrt{34}$ is an exact value. If you change $2\sqrt{34}$ to a decimal number, you get $2\times 5.831 \doteq 11.662$. This is an approximate value rounded to the nearest thousandth.

- 

Solution:

$$\begin{aligned}
\left(10\sqrt{3}\right)^2 &= x^2 + 8^2 \\
10^2 \times \left(\sqrt{3}\right)^2 &= x^2 + 64 \\
100 \times 3 &= x^2 + 64 \\
300 &= x^2 + 64 \\
x^2 &= 300 - 64 \\
\sqrt{x^2} &= \sqrt{236} \\
x &= \sqrt{4 \times 59} \\
&= 2\sqrt{59}
\end{aligned}$$

The value of x is $2\sqrt{59}$.

- 

Solution:

$$\begin{aligned}
x^2 &= \left(\sqrt{6}\right)^2 + \left(3\sqrt{2}\right)^2 \\
x^2 &= 6 + (9 \times 2) \\
x^2 &= 6 + 18 \\
x^2 &= 24 \\
\sqrt{x^2} &= \sqrt{24} \\
x &= \sqrt{4 \times 6} \\
&= 2\sqrt{6}
\end{aligned}$$

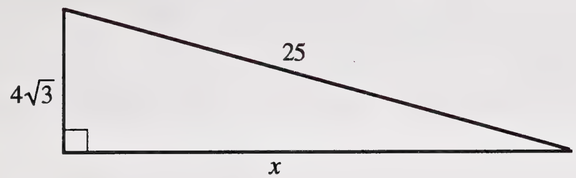
The value of x is $2\sqrt{6}$.

Recall:

$$\begin{aligned}
\left(\sqrt{x}\right)^2 &= \sqrt{x} \times \sqrt{x} \\
&= \sqrt{x \times x} \\
&= \sqrt{x^2} \\
&= x
\end{aligned}$$

$$\begin{aligned}
\left(\sqrt{6}\right)^2 &= \sqrt{6} \times \sqrt{6} \\
&= \sqrt{6 \times 6} \\
&= \sqrt{36} \\
&= 6
\end{aligned}$$

•



Solution:

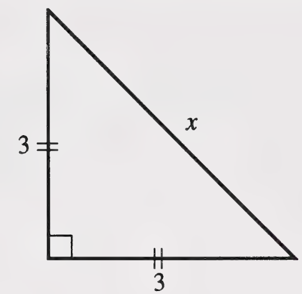
$$25^2 = \left(4\sqrt{3}\right)^2 + x^2$$
$$625 = (16 \times 3) + x^2$$
$$625 = 48 + x^2$$
$$x^2 = 625 - 48$$
$$x^2 = 577$$
$$\sqrt{x^2} = \sqrt{577}$$
$$x = \sqrt{577}$$

The value of x is $\sqrt{577}$.

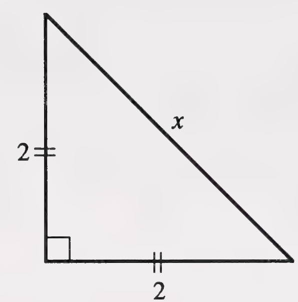
Do either the odd- or even-numbered problems that follow.

1. Find the value of x for the following. Use the simplest form possible.

a.

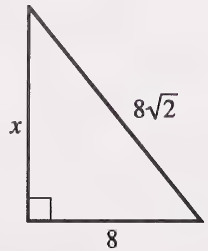


b.

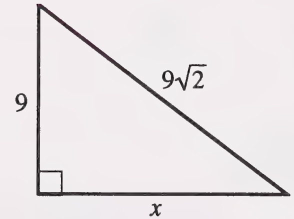


2. Find the value of x for the following.

a.

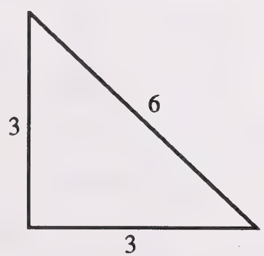


b.

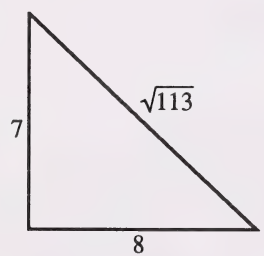


3. Determine whether each of the following is a right triangle. Show why it is or is not a right triangle.

a.

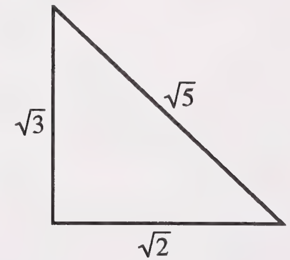


b.

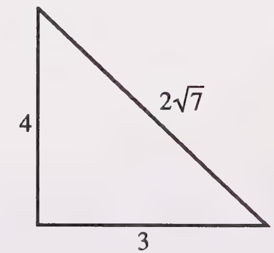


4. Determine whether each of the following is a right triangle. Show why it is or not a right triangle.

a.



b.



5. A staircase has fourteen steps. Each step is 25 cm deep and 20 cm high. Find the length of a banister, which runs alongside the stairway. Express your answer in two ways: as a simplified radical and as a decimal numeral rounded to the nearest tenth of a centimetre.

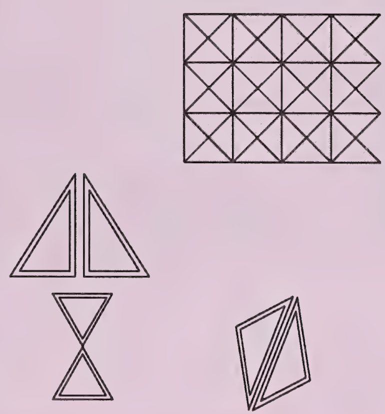
6. Demetri is putting together a border for a rectangular picture. The length of the border is 70 cm and the width is 40 cm. The length of the diagonal is 79 cm. Are the corners of the border right angles?

For solutions to **Extensions**, turn to the **Appendix**, **Topic 1**.

Topic 2 Properties of Similar Triangles

Introduction

Take a few moments to examine your immediate surroundings. With some thought and imagination you will notice similarities among various objects such as bulbs, lamps, picture frames, pieces of furniture, books, and pencils. Similarities are also present if you examine geometric shapes such as quadrilaterals and triangles. In this unit you are going to study the similarities of triangles.



What Lies Ahead

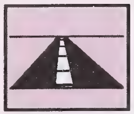
Throughout this topic you will learn to

1. solve problems involving the sum of the angles of a triangle
2. recognize and write the relationship between similar triangles
3. solve problems by using the properties of similar triangles

Now that you know what to expect, turn the page to begin your study of properties of similar triangles.

Exploring Topic 2

Activity 1

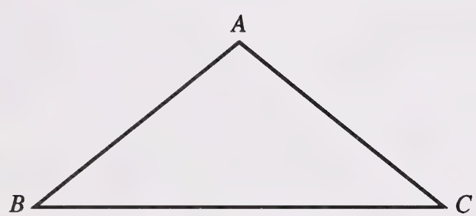


Solve problems involving the sum of the angles of a triangle.

As you examine and use the various aspects related to triangles, there is a basic fact that must be kept in mind at all times.

The sum of the angles of any triangle is 180°.

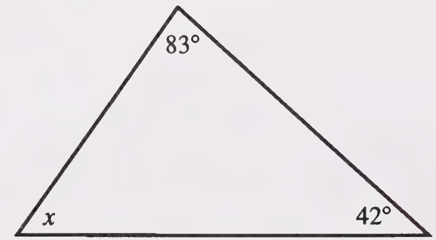
Thus, in ΔABC, $\angle A + \angle B + \angle C = 180°$.



Using this rule you can solve any unknown angle in a given triangle if you know the measures of the other two angles.

Example 1

Find the measure of x.

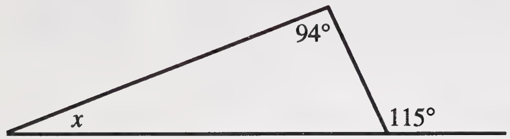


Solution:

$$\begin{aligned} x + 83° + 42° &= 180° \\ x + 125° &= 180° \\ x &= 180° - 125° \\ &= 55° \end{aligned}$$

Example 2

Find the measure of x.



Solution:

The supplementary angle for 115° is $180° - 115° = 65°$.

$$\begin{aligned} x + 94° + 65° &= 180° \\ x + 159° &= 180° \\ x &= 180° - 159° \\ &= 21° \end{aligned}$$

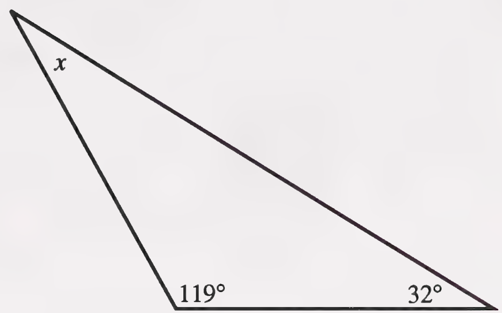
It is now time to try the following exercise..

Do either the odd or the even problems.

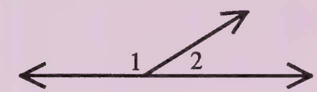
Find the measure of x in each of the following.

1.

2.

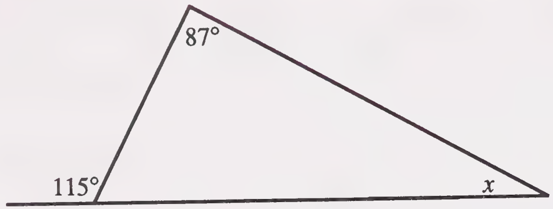


Recall: The sum of supplementary angles is 180°. The following are supplementary, adjacent angles.



$\angle 1 + \angle 2 = 180°$

3.



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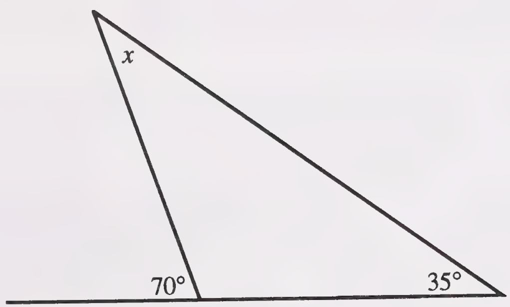
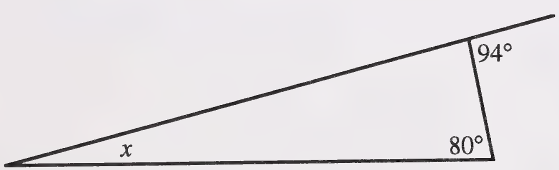
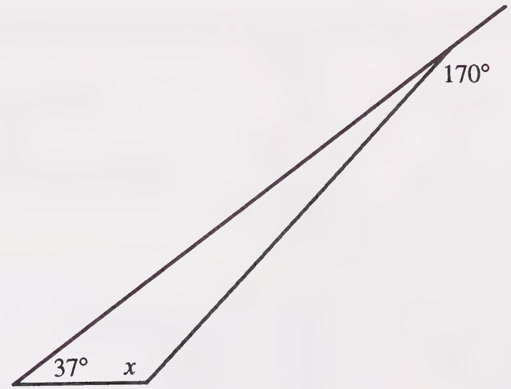


5.

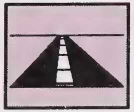


6.



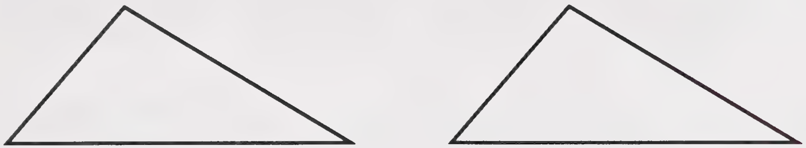
For solutions to **Activity 1**, turn to the **Appendix**, **Topic 2**.

Activity 2

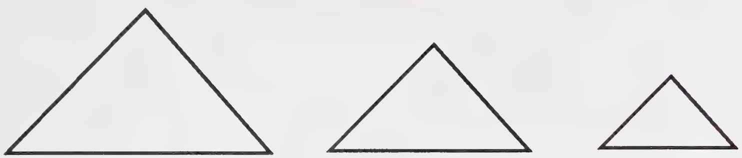


Recognize and write the relationship between similar triangles.

Look at the following triangles. The first pair of triangles are identical in shape and size.



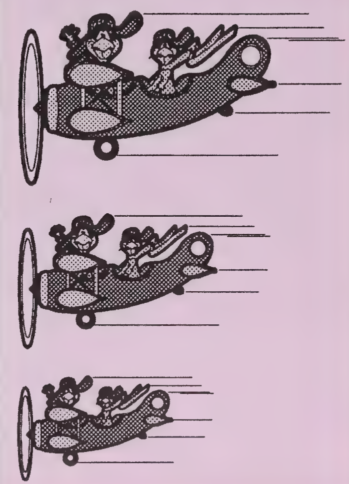
The next set the triangles are not the same in size. Nonetheless, they have a similar appearance.



Upon closer examination of the first pair, you will notice that the triangles are congruent. This means that the triangles are the same. The second set of triangles is similar in shape but different in size. Congruent triangles have congruent corresponding angles and congruent corresponding sides.

Congruent

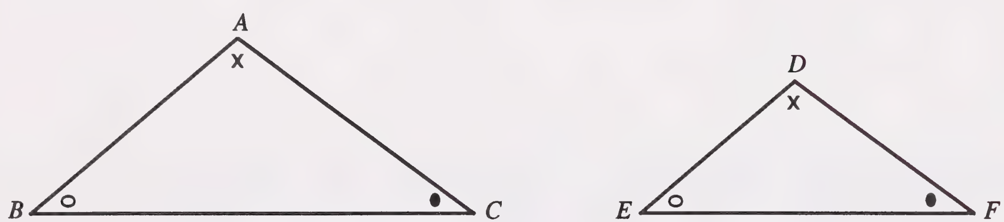
Similar



Two sides are congruent if they are the same length. Two angles are congruent if they are the same size.

Similar triangles have the following important properties:

Given: ΔABC and ΔDEF



Property 1: Corresponding angles are equal in measure.

If $\Delta ABC \sim \Delta DEF$, then $\angle A = \angle D$, $\angle B = \angle E$, and $\angle C = \angle F$.

Property 2: Corresponding sides are proportional.

If $\Delta ABC \sim \Delta DEF$, then $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$.

Referring to these two properties, you can establish rules which apply to situations where similar triangles are needed or used.

Recall: The symbol $\leftrightarrow$ means **corresponds to.**

In ΔABC and ΔDEF,

$\angle A \leftrightarrow \angle D$

$\angle B \leftrightarrow \angle E$

$\angle C \leftrightarrow \angle F$

$\overline{AB} \leftrightarrow \overline{DE}$

$\overline{BC} \leftrightarrow \overline{EF}$

$\overline{AC} \leftrightarrow \overline{DF}$

The sides are line segments; thus, there should be a line over the letters representing the line.

Note: The symbol ~ means **is similar to.**

These proportions could be stated in other ways, such as

$\frac{AB}{AC} = \frac{DE}{DF}$, $\frac{AC}{BC} = \frac{DF}{EF}$, etc.

Rule 1: If the corresponding angles of two triangles are equal, then the triangles are similar.

If you have ΔLMN and ΔOPQ	Conclusion	Which means that in ΔLMN and ΔOPQ
where $\angle L = \angle O$ $\angle M = \angle P$ $\angle N = \angle Q$	$\Delta LMN \sim \Delta OPQ$	$\frac{LM}{OP} = \frac{LN}{OQ} = \frac{MN}{PQ}$

Rule 2: If the corresponding sides of two triangles are proportional, then the triangles are similar.

If you have ΔLMN and ΔOPQ	Conclusion	Which means that in ΔLMN and ΔOPQ
where $\frac{LM}{OP} = \frac{LN}{OQ} = \frac{MN}{PQ}$	$\Delta LMN \sim \Delta OPQ$	$\angle L = \angle O$ $\angle M = \angle P$ $\angle N = \angle Q$

In any similarity relationship, the order used to name the angles and the sides is important and must be standard.

It is sufficient to know that if two pairs of corresponding angles are equal, the two triangles are similar. For example, if $\angle M = \angle P$ and $\angle N = \angle Q$, and the sum of the angles in a triangle is 180°, then $\angle L = \angle O$. Therefore, the triangles are similar.

The following shows corresponding angles.

$\Delta LMN \sim \Delta OPQ$

The following shows corresponding sides.

$\Delta LMN \sim \Delta OPQ$

Example 3

If $\Delta XYZ \sim \Delta RST$, show all the corresponding relationships.

Solution:

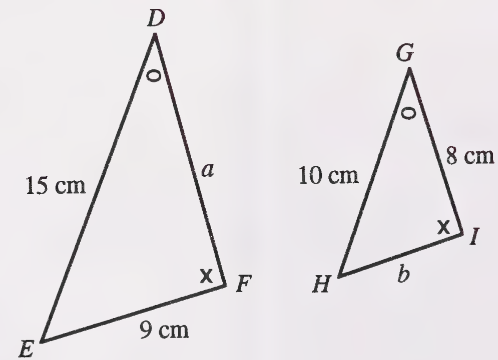
$\angle X$ must correspond to and equal $\angle R$.
$\angle Y$ must correspond to and equal $\angle S$.
$\angle Z$ must correspond to and equal $\angle T$.

$\overline{XY}$ must correspond to $\overline{RS}$.
$\overline{YZ}$ must correspond to $\overline{ST}$.
$\overline{XZ}$ must correspond to $\overline{RT}$.

The properties of similar triangles can be used to find an unknown side.

Example 4

Find the length of a and b.



Solution:

In ΔDEF and ΔGHI, $\angle D = \angle G$ and $\angle F = \angle I$. Thus, $\Delta DEF \sim \Delta GHI$

$$\therefore \frac{HI}{EF} = \frac{GH}{DE} \quad \text{and} \quad \frac{DF}{GI} = \frac{DE}{GH} \quad \text{(Set up the known proportions.)}$$

$$\frac{b}{9} = \frac{10}{15} \qquad \frac{a}{8} = \frac{15}{10} \quad \text{(Substitute.)}$$

$$15b = 90 \qquad 10a = 120 \quad \text{(Cross multiply.)}$$

$$b = 6 \qquad a = 12$$

The length of b is 6 cm, and the length of a is 12 cm.

Now try some questions based on these properties.

Do all of questions 1 to 3; then do the odd- or even-numbered questions (4 to 7).

1. Complete the following.

   a. $\Delta DEF \sim \Delta PQR$

      $\overline{DE}$ is proportional to ______ .

      $\overline{EF}$ is proportional to ______ .

      $\overline{DF}$ is proportional to ______ .

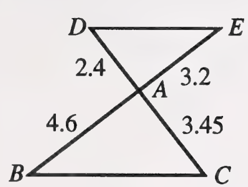
   b. $\Delta ABC \sim \Delta LMN$

      $\angle A =$ ______

      $\angle B =$ ______

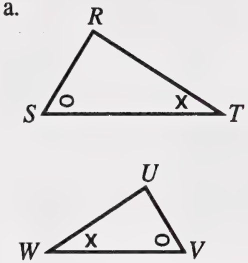
      $\angle C =$ ______

2. Are the following triangles similar? Why?

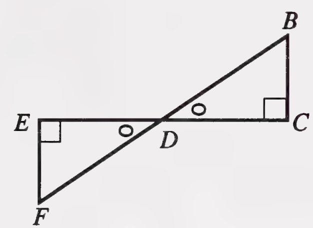


3. State the similarity relationships for each pair of similar triangles.

   a.



   b.

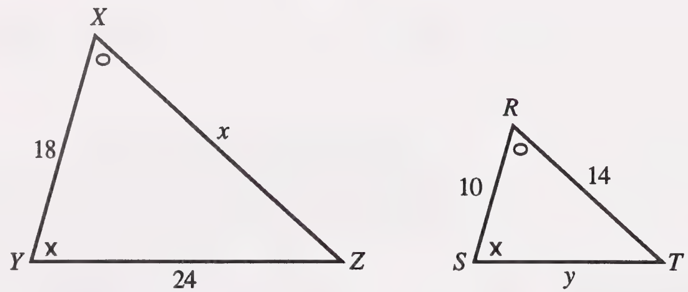


$\Delta ADE \sim \Delta ACB$

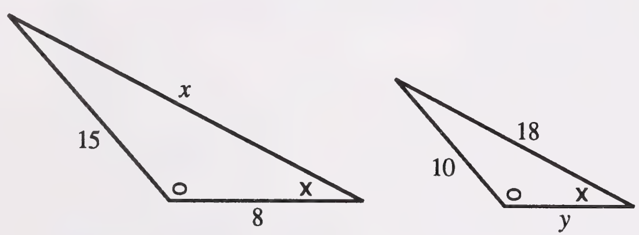
$\therefore \dfrac{AD}{AC} = \dfrac{AE}{BA}$

Two ratios are equal if the cross products are equal.

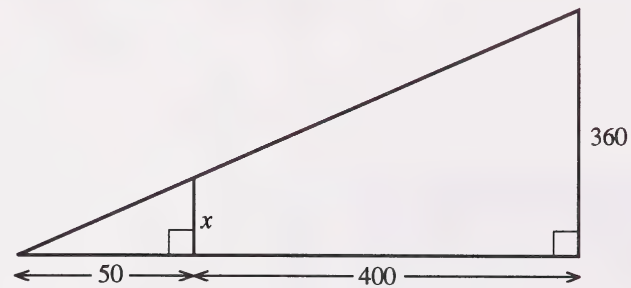
4. Find the value of x and y if $\Delta XYZ \sim \Delta RST$.



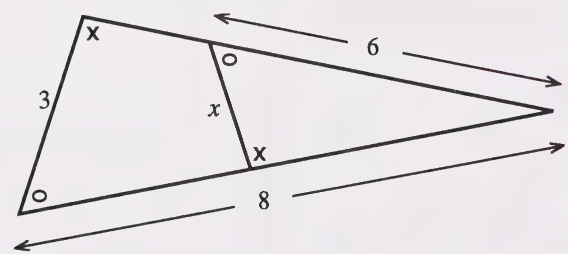
5. Find the value of x and y if the two triangles are similar.



6. Solve for x. Verify your solution.

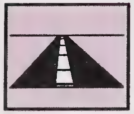


7. Solve for x. Verify your solution.



For solutions to **Activity 2**, turn to the **Appendix**, **Topic 2**.

Activity 3



Solve problems by using the properties of similar triangles.

The properties of similar triangles can be used in an indirect way to find measures, where direct measurement is not practical, to solve many common day-to-day problems. Study the examples given to see how to apply these properties.

The following is an example of indirect measurement using similar triangles.

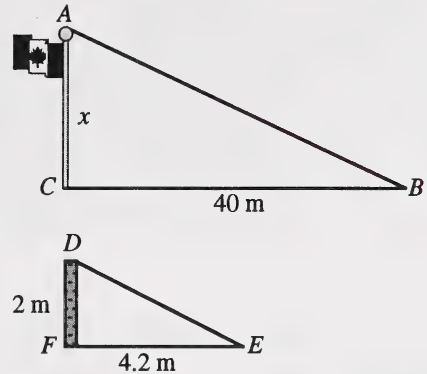
Example 5

At a particular time of day, a flagpole casted a shadow 40 m long, and a fence post 2 m high casted a shadow 4.2 m long. Find the height of the flagpole. Express your answer to the nearest metre.

Solution:

Let x be the height of the flagpole.

Sketch a diagram to show the given information.



In ΔABC and ΔDEF

$\angle C = \angle F$

$= 90°$

$\angle B = \angle E$; thus, $\Delta ABC \sim \Delta DEF$ and $\frac{AC}{DF} = \frac{CB}{FE}$.

The sun's rays strike the ground at the same angle. Therefore, $\angle B = \angle E$.

$$\frac{x}{2} = \frac{40}{4.2}$$

$$4.2x = 80$$

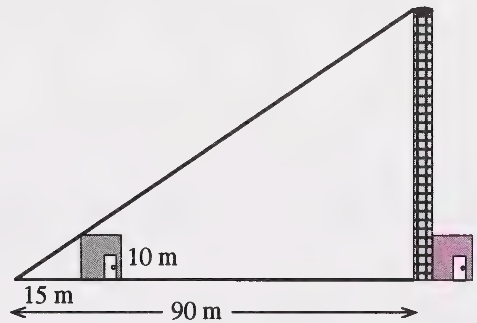
$$x = \frac{80}{4.2}$$

$$\doteq 19.047\,619\,05$$

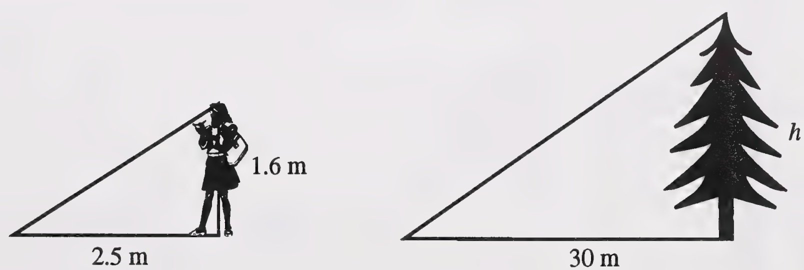
The height of the flagpole to the nearest metre is 19 m.

Now try some practice exercises.

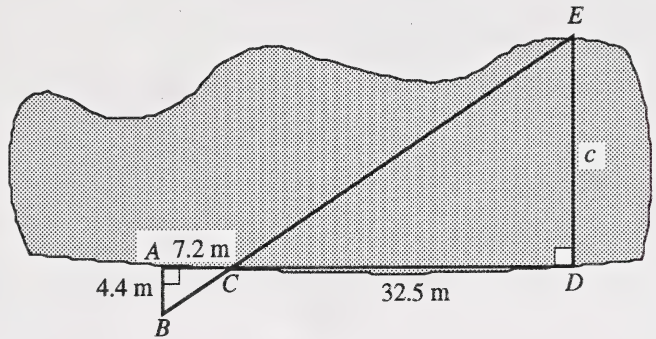
1. Sandra and Krish want to determine the height of an incinerator stack. They line up the edge of the roof of a building 10 m high to the top of the incinerator stack at a point 15 m from the building. They measure this point to be 90 m from the incinerator stack. How high is the incinerator stack. Round your answer to the nearest tenth of a metre.



2. To find the height of a tree, Erin measures the length of her shadow and the length of the tree's shadow. The length of her shadow is 2.5 m, and the length of the tree's shadow is 30 m. If she is 1.6 m tall, how tall is the tree? Round your answer to the nearest tenth of a metre.

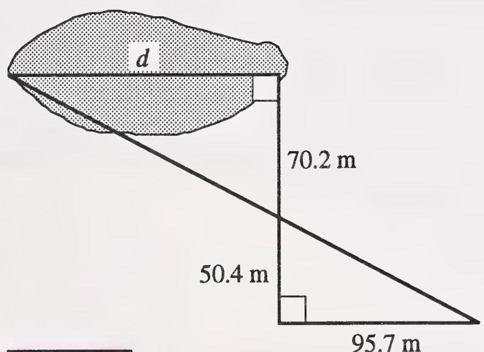


3. To measure the width $\overline{DE}$ of a river, the following measurements were made.



Find the width of the river. Round your answer to the nearest metre.

4. Using the measurements provided, find how far it is across the pond. Round your answer to the nearest metre.



For solutions to **Activity 3**, turn to the **Appendix**, **Topic 2**.

If you require help, do the Extra Help section.

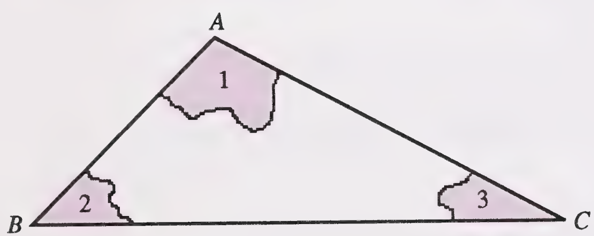
If you want more challenging explorations, do the Extensions section.

} You may decide to do both.

Extra Help

The mathematical fact that the sum of the three angles of any triangle is 180° can be shown in diagram form.

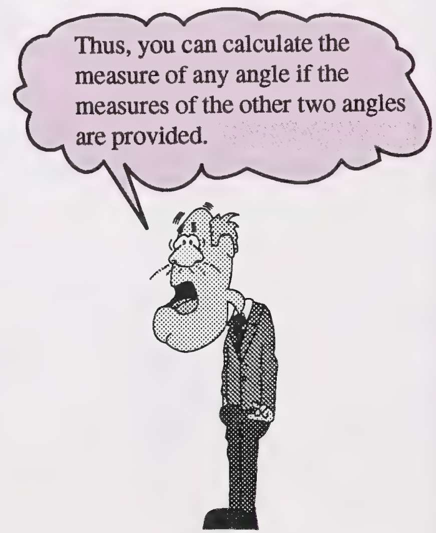
Sketch any triangle as shown, and tear off the vertices. Number these torn off bits as shown.



Next, arrange these pieces as shown.

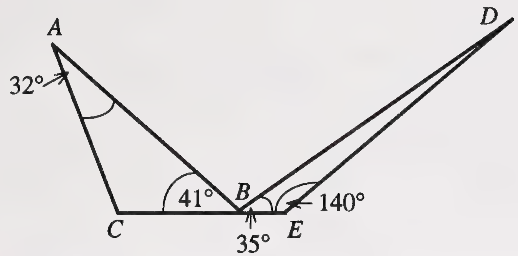
Try this using triangles with different angles.

You can see that the three angles can be fitted to form a straight line. A straight line is really an angle that mesures 180°. Therefore, $\angle 1 + \angle 2 + \angle 3 = 180°$, or $\angle A + \angle B + \angle C = 180°$.



Example 6

Find the measure of $\angle C$ and $\angle D$ in the following diagram.



Solution:

Find the measure of $\angle C$.

$$\begin{aligned} 32°+41°+\angle C &= 180° \\ 73°+\angle C &= 180° \\ \angle C &= 180°-73° \\ &= 107° \end{aligned}$$

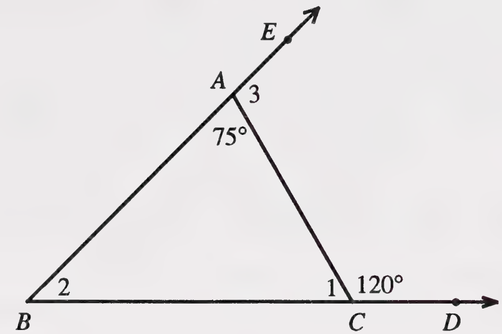
Find the measure of $\angle D$.

$$\begin{aligned} 35°+140°+\angle D &= 180° \\ 175°+\angle D &= 180° \\ \angle D &= 180°-175° \\ &= 5° \end{aligned}$$

Thus, $\angle C = 107°$ and $\angle D = 5°$.

Example 7

Find the measures of $\angle 1$, $\angle 2$, and $\angle 3$.



Solution:

Find the measure of $\angle 1$.

$$\begin{aligned} \angle 1 + 120° &= 180° \\ \angle 1 &= 180°-120° \\ &= 60° \end{aligned}$$

Find the measure of $\angle 2$.

$$\begin{aligned} \angle 2 + 75° + \angle 1 &= 180° \\ \angle 2 + 75° + 60° &= 180° \\ \angle 2 + 135° &= 180° \\ \angle 2 &= 180°-135° \\ &= 45° \end{aligned}$$

Note: $\angle 1$ and $\angle ACD$ are linear angles. This means that they are supplementary angles. The sum of supplementary angles is 180°. The same applies to $\angle 3$ and $\angle BAC$.

Find the measure of $\angle 3$.

$$\begin{aligned}\angle 3 + 75^\circ &= 180^\circ \\ \angle 3 &= 180^\circ - 75^\circ \\ &= 105^\circ\end{aligned}$$

Thus, $\angle 1 = 60^\circ$, $\angle 2 = 45^\circ$, and $\angle 3 = 105^\circ$.

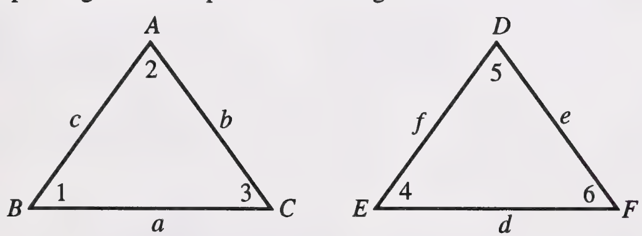
Triangles are similar if the following are true.

- Corresponding angles are congruent. (Numbers can be used to name angles.)
- Corresponding sides are proportional. (Small letters can be used to name the sides.) To be proportional, the ratio of one pair of corresponding sides must equal to the ratio of a second pair of corresponding sides.

Using these basic facts you can solve for the measure of the unknown sides in similar triangles.

Example 8

Show the corresponding relationships for these triangles.



Solution:

If $\Delta ABC \sim \Delta DEF$, then $\angle 1 = \angle 4$, $\angle 2 = \angle 5$, $\angle 3 = \angle 6$, $\frac{c}{f} = \frac{a}{d}$, $\frac{b}{e} = \frac{a}{d}$, and $\frac{b}{e} = \frac{c}{f}$.

Note: Proportional ratios result in equal cross products.

$\frac{3}{4} = \frac{6}{8}$ $\begin{cases} 4 \times 6 = 24 \\ 3 \times 8 = 24 \end{cases}$

$\frac{2}{3} = \frac{10}{15}$ $\begin{cases} 2 \times 15 = 30 \\ 3 \times 10 = 30 \end{cases}$

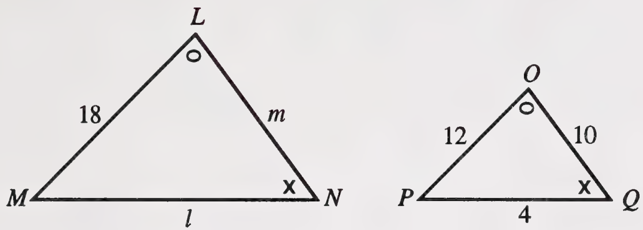
$\frac{15}{18} = \frac{5}{6}$ $\begin{cases} 15 \times 6 = 90 \\ 18 \times 5 = 90 \end{cases}$

Proportions can be done in different ways. These are some which can be used.

$\frac{c}{a} = \frac{f}{d}$, $\frac{a}{b} = \frac{d}{e}$, $\frac{c}{b} = \frac{f}{e}$, etc.

Example 9

Solve for l and m.



Solution:

Solve for l.

$$\frac{18}{12}=\frac{l}{4}$$
$$12l=18\times 4$$
$$12l=72$$
$$\frac{12l}{12}=\frac{72}{12}$$
$$l=6$$

Solve for m.

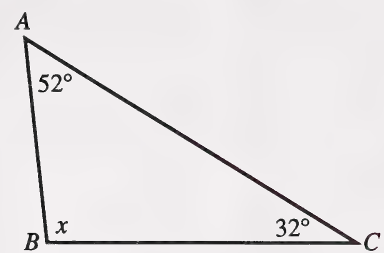
$$\frac{10}{m}=\frac{12}{18}$$
$$12m=18\times 10$$
$$12m=180$$
$$\frac{12m}{12}=\frac{180}{12}$$
$$m=15$$

The value for l is 6, and the value for m is 15.

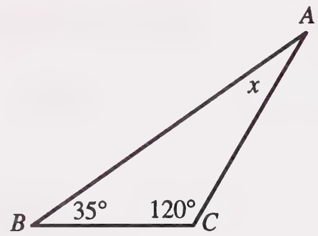
Now try the problems in the next practice exercises. Do all the questions to gain maximum practice and confidence.

1. Solve for the unknown angles in the following.

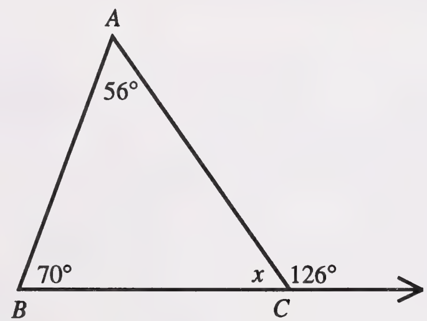
a.



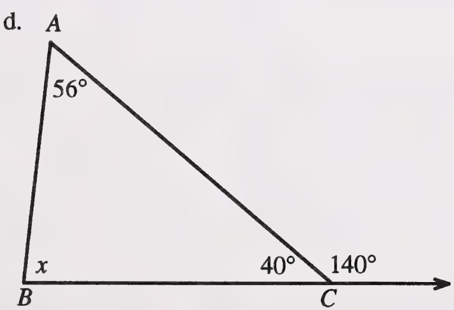
b.



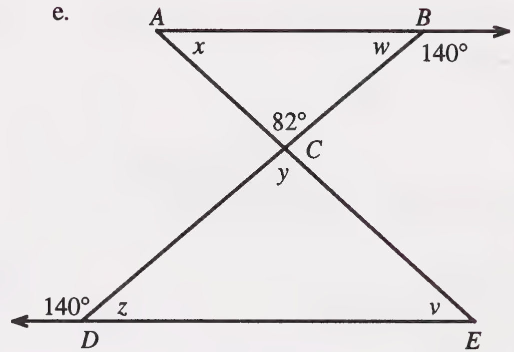
c.



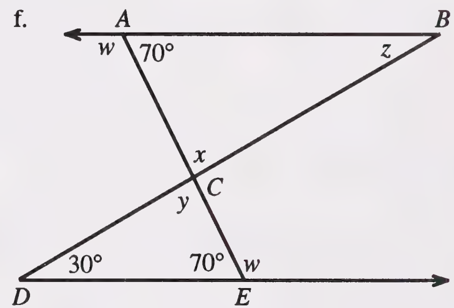
d.



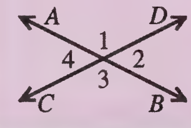
e.



f.



Recall: When two lines intersect, the opposite angles are equal.

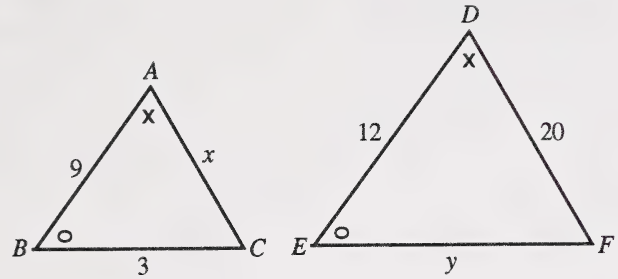


$\angle 1 = \angle 3$

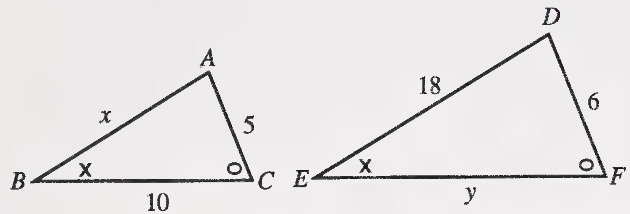
$\angle 2 = \angle 4$

2. Each pair of triangles is similar. Solve for the unknown value or values in each case.

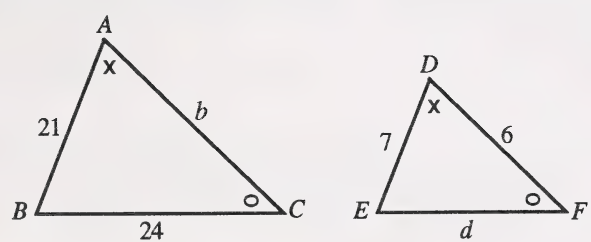
a.



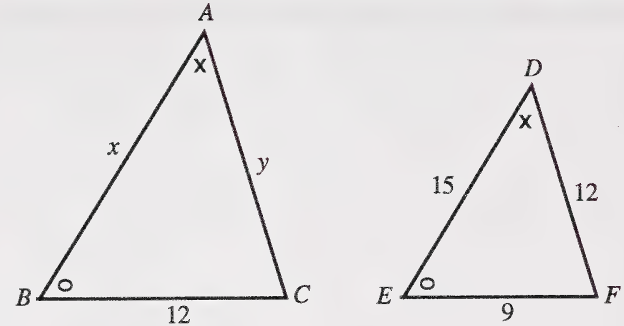
b.



c.



d.



3. A tree casts a shadow 15.5 m long. At the same time Shanti casts a shadow 3.6 m long. If Shanti is 1.8 m tall, find the height of the tree. (Hint: Sketch a diagram to begin.)

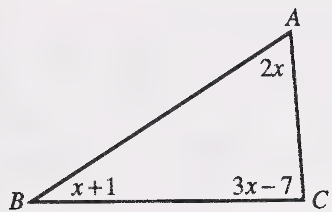
For solutions to **Extra Help**, turn to the **Appendix**, **Topic 2**.

Extensions

Sometimes the measures of the angles of a triangle are shown as algebraic expressions. To find the measures of these angles you still use the basic fact that the sum of the three angles in any triangle is 180°.

Example 10

Give the measure of each angle in ΔABC.



Solution:

$$2x+x+1+3x-7=180$$
$$6x-6=180$$
$$\frac{6x}{6}=\frac{186}{6}$$
$$x=31$$

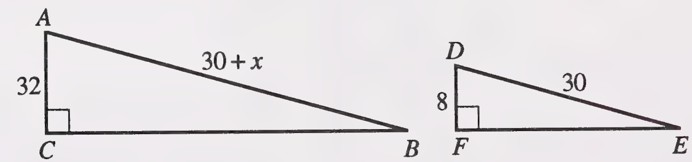
$\angle A=2(31)$ $=62°$ $\angle B=31+1$ $=32°$ $\angle C=3(31)-7$ $=86°$

Since the measures of the angles of a triangle can be given as algebraic expressions, it would follow that the sides of a triangle can be shown using similar algebraic expressions.

To solve for the unknown measures, use the basic fact that in similar triangles the corresponding sides are proportional.

Example 11

Find the length of $\overline{AB}$, $\overline{CB}$, and $\overline{FE}$ if $\Delta ACB \sim \Delta DEF$.



Solution:

First, find the value of x.

$$\frac{AC}{DF}=\frac{AB}{DE}$$
$$\frac{32}{8}=\frac{30+x}{30}$$
$$8(x+30)=32\times 30$$
$$8x+240=960$$
$$8x=960-240$$
$$8x=720$$
$$\frac{8x}{8}=\frac{720}{8}$$
$$x=90$$

Now find the length of $\overline{AB}$.

$$\begin{aligned} AB &= 30 + x \\ &= 30 + 90 \\ &= 120 \text{ units} \end{aligned}$$

Find the length of $\overline{CB}$.

$$\begin{aligned} CB^2 &= AB^2 - AC^2 \\ CB^2 &= 120^2 - 32^2 \\ CB^2 &= 14\,400 - 1024 \\ CB^2 &= 13\,376 \\ \sqrt{CB^2} &= \sqrt{13\,376} \\ CB &\doteq 115.654\,6584 \\ CB &\doteq 116 \text{ units} \quad \text{(to the nearest whole number)} \end{aligned}$$

Finally, find the value of $\overline{FE}$.

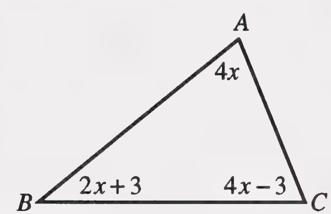
$$\begin{aligned} FE^2 &= DE^2 + DF^2 \\ FE^2 &= 30^2 - 8^2 \\ FE^2 &= 900 - 64 \\ FE^2 &= 836 \\ \sqrt{FE^2} &= \sqrt{836} \\ FE &\doteq 28.913\,664\,59 \\ FE &\doteq 29 \text{ units} \quad \text{(to the nearest whole number)} \end{aligned}$$

Thus, $AB = 120$ units, $CB \doteq 116$ units, and $FE \doteq 29$ units.

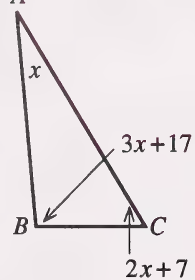
Apply your skills in the following questions. If the questions seem somewhat difficult, do not give up. Keep thinking and trying. Try as many as you can.

1. Find the measure of all the angles in each triangle.

a.

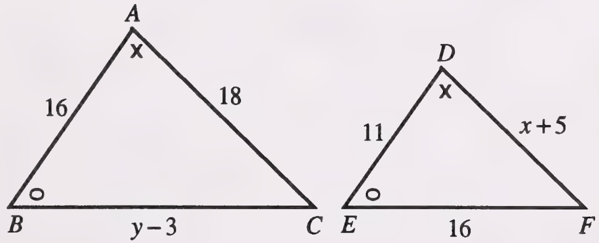


b.

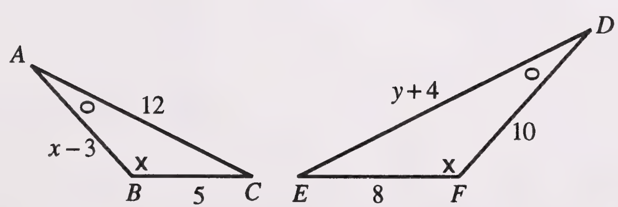


2. Find the length of the side represented by an algebraic expression. If necessary, round off to the nearest tenth.

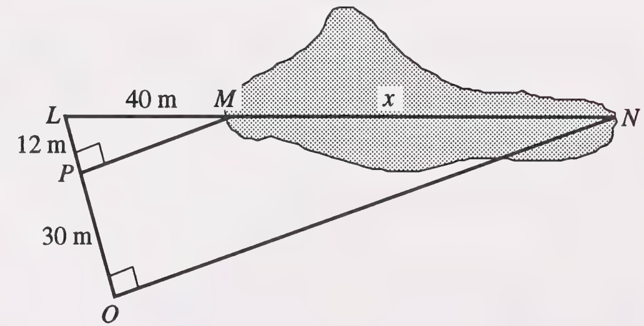
a.



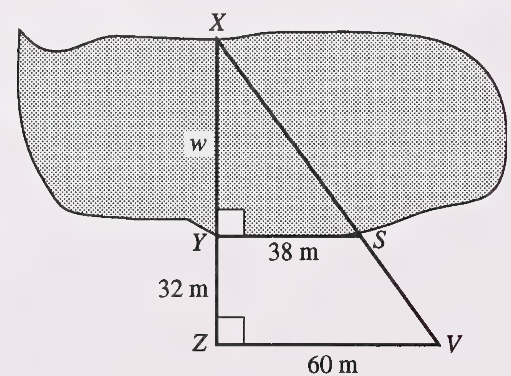
b.

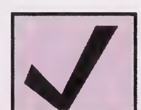


3. a. Calculate x, which is the length of the lake. (Hint: The length of $\overline{LN}$ is $(40+x)$ metres.)



b. Calculate w, which is the the width of the river. Round your answer to the nearest metre?





For solutions to **Extensions**, turn to the **Appendix**, **Topic 2**.

Topic 3 Developing and Finding Trigonometric Ratios

Introduction

Have you ever used a dictionary to define a word? Have you ever put words together to convey ideas? As you talk or write, words and ideas are used constantly. If you are to communicate effectively in trigonometry, there must be an agreement on the meaning of the terms used in this topic. You must learn the language of trigonometry so that you can clearly communicate and understand the concepts used.

What Lies Ahead

Throughout this topic you will learn to

1. define the terms opposite side, adjacent side, and hypotenuse

2. use the properties of similar triangles to develop the tangent, sine, and cosine ratios

3. use a calculator to determine the trigonometric ratio given the measure of an angle, and to determine the measure of an angle given a particular ratio

4. determine the sine, cosine, and tangent ratios within right triangles, given the measures of any two sides

Now that you know what to expect, turn the page to begin your study of developing and finding trigonometric ratios.

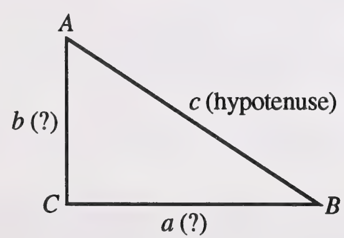
Exploring Topic 3

Activity 1

Define the terms opposite side, adjacent side, and hypotenuse.

Everything does not go by the same name. Different names may be applied to the same thing depending on the situation. You learned earlier that the sides of a triangle are named by using a small letter which corresponds to the letter representing the opposite angle of that particular side.

You already know that the side opposite the right angle in a right-angle triangle is commonly called the **hypotenuse**. How about the other two sides? Do they have names as well?



To name these sides, choose one of the acute angles as a starting point. Such an angle is called the reference angle. The reference angle might be called $\angle A$, $\angle B$, or any other symbol that is chosen to represent that angle.

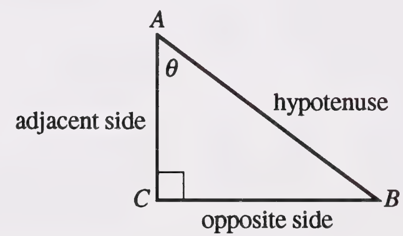
The side directly across from the reference angle is called the **opposite side**. The side next to the reference angle that is not the hypotenuse is called the **adjacent side**. Therefore, the sides may be called hypotenuse, opposite, and adjacent rather than a, b, and c. Study the examples that follow.

Example 1

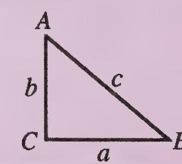
Label the sides of ΔABC opposite, adjacent, and hypotenuse in reference to the following angles.

- $\angle A$ or $\angle\theta$

Solution:

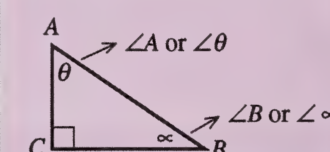


Recall: Labelling triangles



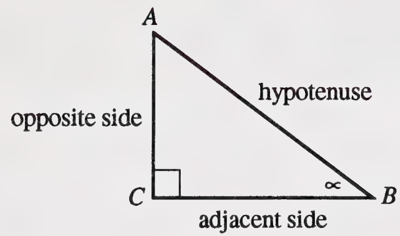
Note: The symbols θ (theta) and $\propto$ (alpha) are letters of the Greek alphabet. They are used to name angles.

Note: Ways of showing the reference angle are as follows:



- $\angle B$ or $\angle \propto$

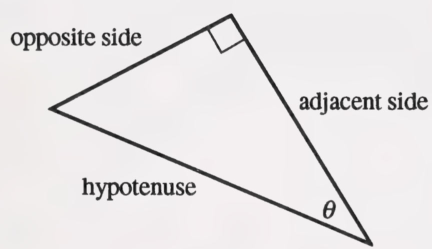
Solution:



Example 2

With reference to $\angle\theta$, label the sides of the following triangle.

Solution:

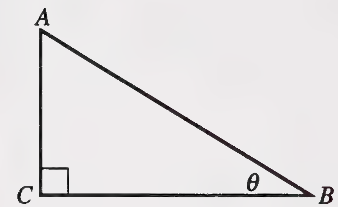


Do the following exercise to practice what you have learned.

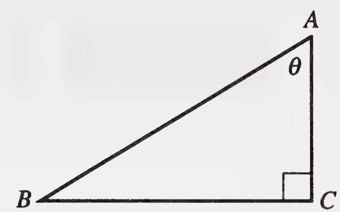
Do either question 1 or question 2.

1. Name the side opposite and the side adjacent to $\angle\theta$ in the following triangles.

a.

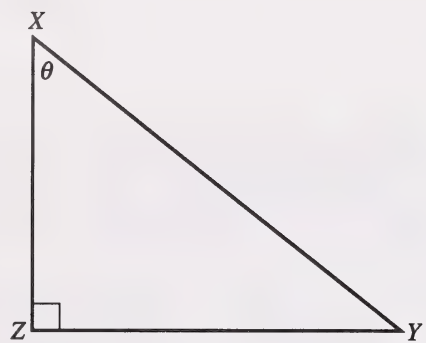


b.

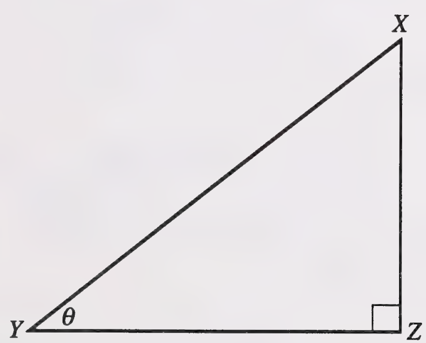


2. Name the side opposite and the side adjacent to $\angle\theta$ in the following triangles.

a.



b.



For solutions to **Activity 1**, turn to the **Appendix**, **Topic 3**.

Activity 2

Use the properties of similar triangles to develop the tangent, sine, and cosine ratios.

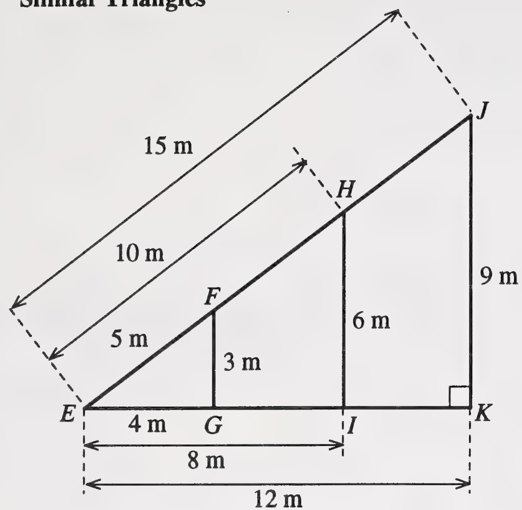
You may study this activity by doing either **Part A** or **Part B**. **Part A** teaches this activity by audiotape, while **Part B** teaches it through the print mode. Whichever way you choose to study this activity, do the questions at the end of Activity 2 in the print pathway.

Part A

Audio Activity

Insert the tape entitled *Math 10, Unit 7 – Ratios* into your tape recorder. Turn on your tape recorder and follow the instructions on the tape.

1 Similar Triangles



$$\frac{FG}{EG}=\frac{3}{4}$$
$$=0.75$$

$$\frac{HI}{EI}=\frac{6}{8}$$
$$=0.75$$

Complete:

$$\frac{JK}{EK}= \_\_\_\_$$
$$= \_\_\_\_$$

2 Ratios of Opposite Side to Adjacent Side

$$\frac{FG}{EG}=\frac{HI}{EI}=\frac{JK}{EK}=0.75$$

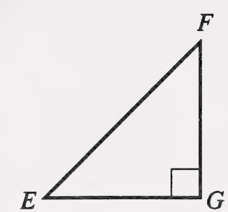
Does the ratio depend on the size of the triangle or the size of the angle?

3 Tangent Ratio

If $\angle E$ is an acute angle in a right triangle,

then $\tan E=\dfrac{\text{length of side opposite } \angle E}{\text{length of side adjacent to } \angle E}$.

4 Each Acute Angle Has a Different Ratio



$$\tan E=\frac{\text{side opposite}}{\text{side adjacent}}$$
$$=\frac{FG}{EG}$$

$$\tan F=\frac{\text{side opposite}}{\text{side adjacent}}$$
$$=\frac{EG}{FG}$$

5 Ratios of Side Opposite Angle E to the Hypotenuse

$$\frac{FG}{EF} = \frac{3}{5} = 0.6 \qquad \frac{HI}{EH} = \frac{6}{10} = 0.6$$

Complete:

$$\frac{JK}{EJ} = \_\_\_ = \_\_\_$$

$$\frac{FG}{EF} = \frac{HI}{EH} = \frac{JK}{EJ} = 0.6$$

6 Sine Ratio

If $\angle E$ is an acute angle in a right triangle,

then $\sin E = \dfrac{\text{length of side opposite } \angle E}{\text{length of the hypotenuse}}$.

7 Summary of Ratios of Adjacent Side to Hypotenuse

$$\frac{EG}{EF} = \frac{4}{5} = 0.8$$

Complete:

$$\frac{EI}{EH} = \_\_\_ = \_\_\_ \qquad \frac{EK}{EJ} = \_\_\_ = \_\_\_$$

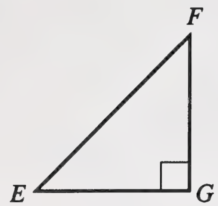
$$\frac{EG}{EF} = \frac{EI}{EH} = \frac{EK}{EJ} = 0.8$$

8 Cosine Ratio

If $\angle E$ is an acute angle in a right triangle,

then $\cos E = \dfrac{\text{length of side adjacent } \angle E}{\text{length of the hypotenuse}}$.

9 Complementary Ratios

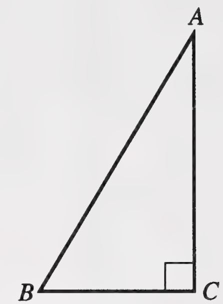


$\sin E = \frac{FG}{EF}$ $\qquad$ $\sin F = \frac{EG}{EF}$

$\cos E = \frac{EG}{EF}$ $\qquad$ $\cos F = \frac{FG}{EF}$

Note that $\sin E = \cos F$ and $\cos E = \sin F$.

10 Summary

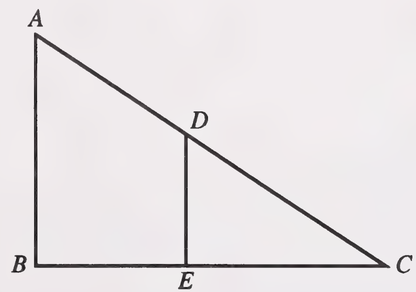


$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{BC}{AB} \qquad \cos A = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{AC}{AB} \qquad \tan A = \frac{\text{opposite}}{\text{adjacent}} = \frac{BC}{AC}$$

$$\sin B = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{AC}{AB} \qquad \cos B = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{BC}{AB} \qquad \tan B = \frac{\text{opposite}}{\text{adjacent}} = \frac{AC}{BC}$$

Part B

Now that you know how to label the sides of a right triangle using hypotenuse, side opposite, and side adjacent, apply this special labelling process to your knowledge of similar triangles.



$\overline{AB} \leftrightarrow \overline{DE}$

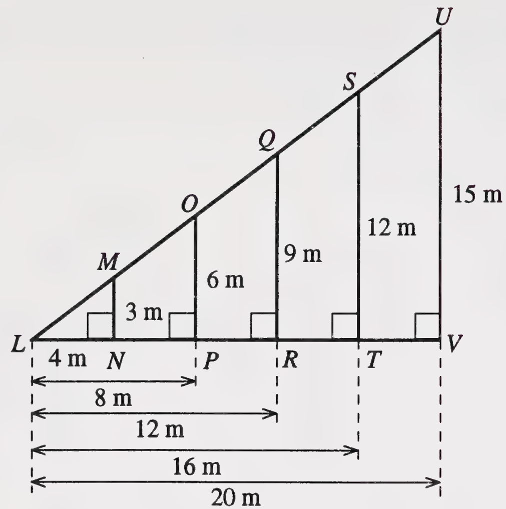
$\overline{DC} \leftrightarrow \overline{AC}$

Which side corresponds with $\overline{BC}$?

$\overline{BC} \leftrightarrow \overline{EC}$

To solve for distance indirectly, you used two similar triangles knowing that the corresponding sides are proportional. To save time and space, a method has been invented which uses only one triangle to solve problems involving indirect measurement. The diagram that follows shows five similar right triangles which are placed one on top of the other by placing the largest one down first and the smallest one last. The triangles are similar since $\angle L$ is common and each has one right angle. To check if the triangles are similar, set up the ratios of the corresponding sides. To do this use the sides opposite $\angle L$ and the sides adjacent $\angle L$.

Always be sure to use the corresponding sides in the proportion.



$\frac{MN}{LN} = \frac{3\text{ m}}{4\text{ m}}$
$= 0.75$

$\frac{OP}{LP} = \frac{6\text{ m}}{8\text{ m}}$
$= 0.75$

$\frac{QR}{LR} = \frac{9\text{ m}}{12\text{ m}}$
$= 0.75$

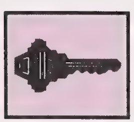
$\frac{ST}{LT} = \frac{12\text{ m}}{16\text{ m}}$
$= 0.75$

$\frac{UV}{LV} = \frac{15\text{ m}}{20\text{ m}}$
$= 0.75$

Recall: To change a fraction to a decimal, the top number is divided by the bottom number.

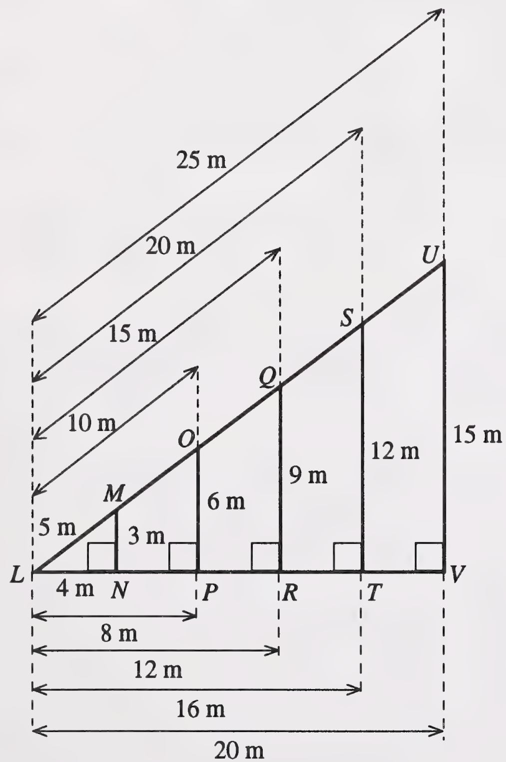
Thus, $\frac{MN}{LN} = \frac{OP}{LP} = \frac{QR}{LR} = \frac{ST}{LT} = \frac{UV}{LV} = 0.75$.

The ratios are all equivalent. These ratios depend only on the measure of $\angle L$, and not on the size of the triangle. This ratio is called the tangent of $\angle L$. It is often written as $\tan \angle L$ or just $\tan L$.



If $\angle L$ is an acute angle in a right triangle, then $\tan L = \frac{\text{length of side opposite } \angle L}{\text{length of side adjacent to } \angle L}$.

Since every triangle has three sides, additional ratios involving the hypotenuse have been developed.

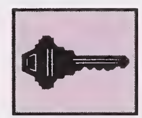


$$\frac{MN}{LM}=\frac{3\text{ m}}{5\text{ m}}=0.6 \qquad \frac{OP}{LO}=\frac{6\text{ m}}{10\text{ m}}=0.6 \qquad \frac{QR}{LQ}=\frac{9\text{ m}}{15\text{ m}}=0.6$$

$$\frac{ST}{LS}=\frac{12\text{ m}}{20\text{ m}}=0.6 \qquad \frac{UV}{LU}=\frac{15\text{ m}}{25\text{ m}}=0.6$$

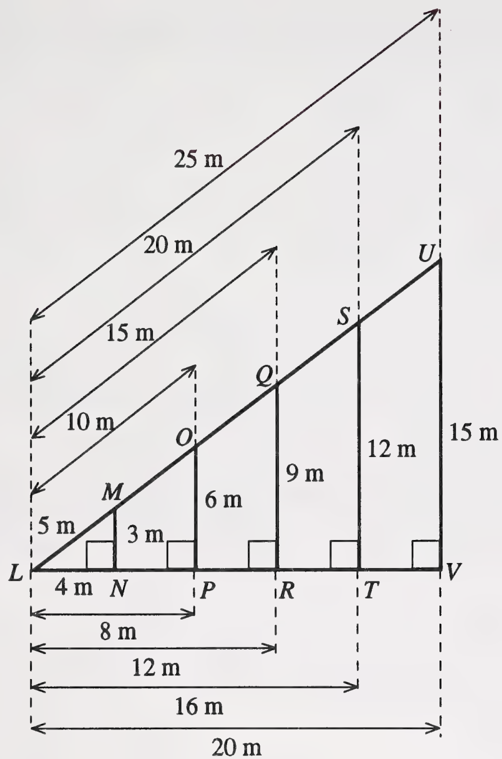
Therefore, $\frac{MN}{LM}=\frac{OP}{LO}=\frac{QR}{LQ}=\frac{ST}{LS}=\frac{UV}{LU}=0.6$.

The ratios are equivalent. This ratio is called the sine of $\angle L$, and is written as $\sin L$.



If $\angle L$ is an acute angle in a right triangle, then

$$\sin L=\frac{\text{length of side opposite }\angle L}{\text{length of the hypotenuse}}.$$



$$\frac{LN}{LM}=\frac{4\text{ m}}{5\text{ m}}$$
$$=0.8$$

$$\frac{LP}{LO}=\frac{8\text{ m}}{10\text{ m}}$$
$$=0.8$$

$$\frac{LR}{LQ}=\frac{12\text{ m}}{15\text{ m}}$$
$$=0.8$$

$$\frac{LT}{LS}=\frac{16\text{ m}}{20\text{ m}}$$
$$=0.8$$

$$\frac{LV}{LU}=\frac{20\text{ m}}{25\text{ m}}$$
$$=0.8$$

Therefore, $\frac{LN}{LM}=\frac{LP}{LO}=\frac{LR}{LQ}=\frac{LT}{LS}=\frac{LV}{LU}=0.8$.

Once again, all the ratios are equivalent. This ratio is called the cosine of $\angle L$ and the ratio is written as $\cos L$. The sine and the cosine ratios are often referred to as complementary ratios.

If $\angle L$ is an acute angle in a right triangle, then

$$\cos L=\frac{\text{length of side adjacent to }\angle L}{\text{length of hypotenuse}}.$$

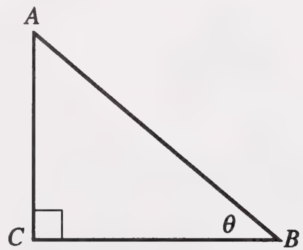
Tangent, sine, and cosine ratios are often referred to as the primary trigonometric ratios.

It is now time to apply the skills you learned thus far to answer some questions on your own. Proceed to the problems that follow. If you have trouble answering these correctly, go to the **Extra Help** section. For more challenge, you may choose to do the **Extensions** section.

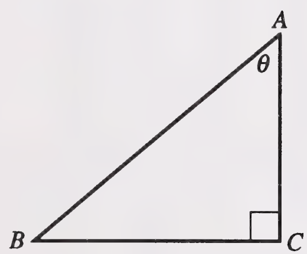
Do either the odd- or the even-numbered questions.

1. Give the three primary trigonometric ratios (tan, sin, and cos) for θ in ΔABC.

a.

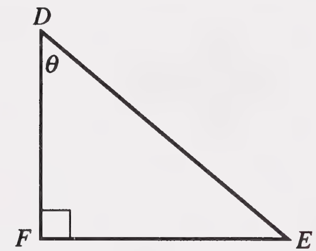


b.

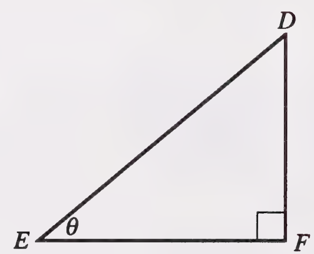


2. Give the three primary trigonometric ratios for θ in ΔDEF.

a.



b.



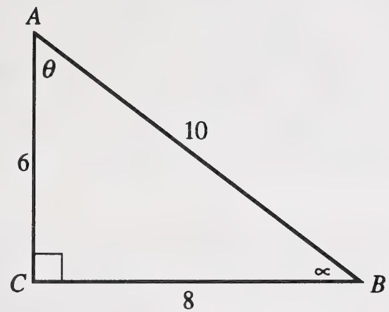
Remember:

$$\text{sine} = \frac{\text{side opposite}}{\text{hypotenuse}}$$

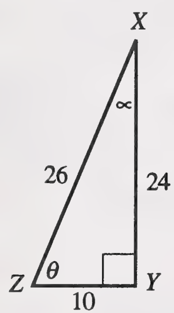
$$\text{cosine} = \frac{\text{side adjacent}}{\text{hypotenuse}}$$

$$\text{tangent} = \frac{\text{side opposite}}{\text{side adjacent}}$$

3. Using the provided values, give the primary trigonometric ratios for θ and $\propto$. Express your answer in fraction and decimal form. Round your answer to two decimal places if they do not work evenly.



4. Using the provided values, give the primary trigonometric ratios for θ and $\propto$. Express your answers in fraction and decimal form. Round your answer to two decimal places if they do not work evenly.



For solutions to **Activity 2**, turn to the **Appendix**, **Topic 3**.

The primary trigonometric ratios are sine, cosine, and tangent.

Note: The symbol θ is the Greek letter **theta**, and the symbol $\propto$ is the Greek letter **alpha**.

Activity 3

Use a calculator to determine the trigonometric ratio given the measure of an angle, and to determine the measure of an angle given a particular ratio.

A long time ago scholars of trigonometry calculated the primary trigonometric ratios for every acute angle. These ratios are decimal values rounded to four decimal places and were organized into tables. Such tables can be found somewhere in almost any mathematics textbook. Today scientific calculators are used to find the primary trigonometric ratios for any angle.

Example 3

Use a calculator to find the primary trigonometric ratios for an angle that is 28°. The calculator should be in degree mode. Your calculator may not work as shown in this unit. Consult your manual if you do not get the answers given by following the procedures.

Solution:

sin 28°

Enter	Display
28	28
$\boxed{\text{sin}}$	0.469471562

$\therefore \sin 28° \doteq 0.4695$ (rounded to four decimal places)

cos 28°

Enter	Display
28	28
$\boxed{\text{cos}}$	0.882947592

$\therefore \cos 28° \doteq 0.8829$ (rounded to four decimal places)

tan 28°

Enter	Display
28	28
$\boxed{\text{tan}}$	0.531709431

$\therefore \tan 28° \doteq 0.5317$ (rounded to four decimal places)

Previously, you learned how the three primary trigonometric ratios for the acute angles can be found using a calculator. Now you will learn to do the opposite or reverse of that. This means that you will use a calculator to find the measure of an angle when one of the trigonometric ratios is given. Rounding is usually needed. The measure of the required angle is usually rounded to the nearest degree. Study the following example.

Example 4

Use a calculator to find $\angle C$ if $\tan C = \frac{3}{5}$. Round to the nearest degree.

Solution:

Enter	Display
3	3
$\boxed{\div}$	3
5	5
$\boxed{=}$	0.6
$\boxed{\text{INV}}$	0.6
$\boxed{\text{tan}}$	30.96375653

Therefore, $\angle C \doteq 31°$.

Example 5

Use a calculator to find $\angle C$ if $\cos C = \frac{5}{6}$. Round to the nearest degree.

Solution:

Enter	Display
5	5
$\boxed{\div}$	5
6	6
$\boxed{=}$	0.833333333
$\boxed{\text{INV}}$	0.833333333
$\boxed{\text{cos}}$	33.55730976

Therefore, $\angle C \doteq 34°$.

Now try some of these on your own. Do the following exercise.

Recall: To change any fraction to a decimal, divide the denominator into the numerator. For example,

$$\frac{2}{5} = 2 \div 5$$
$$= 0.4$$

On some calculators

$\boxed{\text{INV}} = \boxed{\text{2nd F}}$

Do all of questions 1, 2, and 3, and at least three of questions 4, 5, 6, 7, and 8. Round the ratios to four decimal places and the angles to the nearest degree.

1. a. Find tan B when $\angle B$ is 11°, 21°, 31°, 41°, 51°, and 61°.

   b. As $\angle B$ increases in measure, what happens to the tangent values?

2. a. Find sin B when $\angle B$ is 5°, 10°, 15°, 20°, 25°, and 30°.

   b. As $\angle B$ increases in measure, what happens to the sine values?

3. a. Find cos B when $\angle B$ is 14°, 24°, 27°, 37°, 41°, and 51°.

   b. As $\angle B$ increases in measure, what happens to the cosine values?

4. Find $\angle A$ if $\tan A = \frac{11}{15}$.

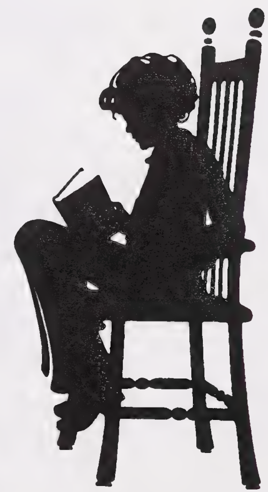
5. Find $\angle B$ if $\cos B = \frac{16}{23}$.

6. Find $\angle A$ if $\sin A = \frac{24}{54}$.

7. Find $\angle C$ if $\sin C = \frac{2}{7}$.

8. Find $\angle B$ if $\tan B = \frac{7}{3}$.

For solutions to **Activity 3**, turn to the **Appendix**, **Topic 3**.



Recall: The primary trigonometric ratios are as follows:

$$\tan = \frac{\text{side opposite}}{\text{side adjacent}}$$

$$\sin = \frac{\text{side opposite}}{\text{hypotenuse}}$$

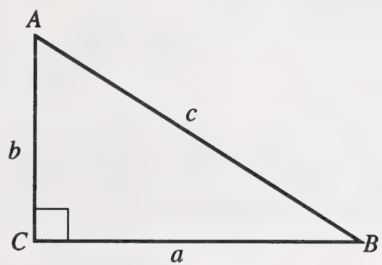
$$\cos = \frac{\text{side adjacent}}{\text{hypotenuse}}$$

Activity 4

Determine the sine, cosine, and tangent ratios within right triangles, given the measures of any two sides.

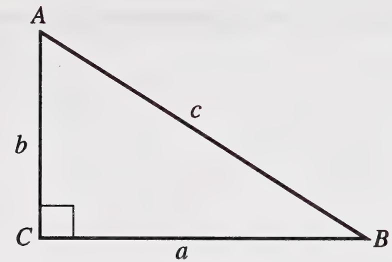
You have already learned that for any acute angle in a right triangle, the measures for particular sides are needed to find the primary trigonometric ratios.

To find the tangent ratio, the measures needed are the side opposite the reference angle and the side adjacent to the reference angle.



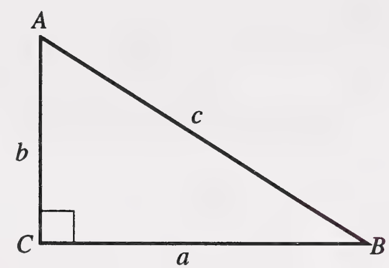
In ΔABC, $\tan A = \frac{a}{b}$ and $\tan B = \frac{b}{a}$.

To find the sine ratio, the measures needed are the side opposite the reference angle and the hypotenuse.



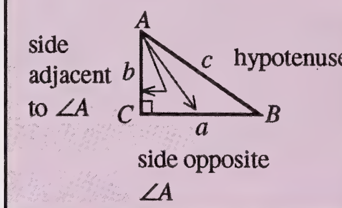
In ΔABC, $\sin A = \frac{a}{c}$ and $\sin B = \frac{b}{c}$.

To find the cosine ratio, the measures needed are the side adjacent to the reference angle and the hypotenuse.



In ΔABC, $\cos A = \frac{b}{c}$ and $\cos B = \frac{a}{c}$.

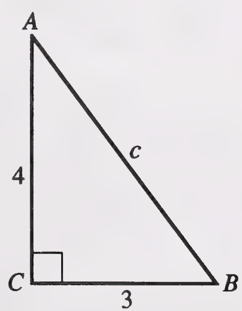
Recall:



By now you should be able to conclude that whenever the measures of any **two sides** of a right triangle are given, the primary trigonometric ratios can be found. Keep in mind that the ratios can be expressed in fractional form and in decimal form. Study the following examples.

Example 6

Find the three primary trigonometric ratios for $\angle A$ and $\angle B$ in the given right triangle. Leave the ratios in simplest fraction form.



Solution:

First, find the measure of the hypotenuse.

$$c^2 = a^2 + b^2$$
$$c^2 = 3^2 + 4^2$$
$$c^2 = 9 + 16$$
$$c^2 = 25$$
$$\sqrt{c^2} = \sqrt{25}$$
$$c = 5$$

The ratios for $\angle A$ are as follows:

$\sin A = \frac{3}{5}$ $\cos A = \frac{4}{5}$

$\tan A = \frac{3}{4}$

The ratios for $\angle B$ are as follows:

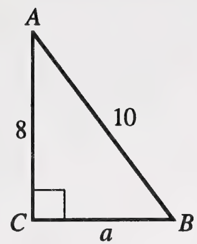
$\sin B = \frac{4}{5}$ $\cos B = \frac{3}{5}$

$\tan B = \frac{4}{3}$

Recall: $c^2 = a^2 + b^2$

Example 7

Find the three primary trigonometric ratios for $\angle A$ and for $\angle B$ in the given right triangle. Leave the ratios in simplest fraction form.



Solution:

First, find the measure of a.

$$\begin{aligned} c^2 &= a^2 + b^2 \\ 10^2 &= a^2 + 8^2 \\ 100 &= a^2 + 64 \\ a^2 &= 100 - 64 \\ a^2 &= 36 \\ \sqrt{a^2} &= \sqrt{36} \\ a &= 6 \end{aligned}$$

The ratios for $\angle A$ are as follows:

$$\sin A = \frac{6}{10} = \frac{3}{5} \qquad \cos A = \frac{8}{10} = \frac{4}{5}$$

$$\tan A = \frac{6}{8} = \frac{3}{4}$$

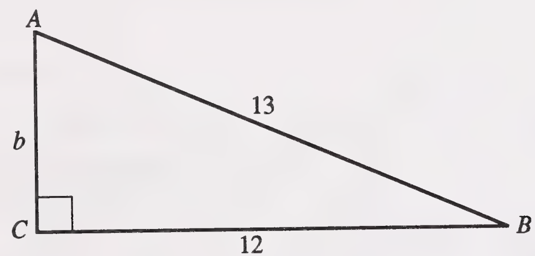
The ratios for $\angle B$ are as follows:

$$\sin B = \frac{8}{10} = \frac{4}{5} \qquad \cos B = \frac{6}{10} = \frac{3}{5}$$

$$\tan B = \frac{8}{6} = \frac{4}{3}$$

Example 8

Use a calculator to find the three primary trigonometric ratios for $\angle A$ and $\angle B$ in the given right triangle. Express each value in decimal form. Round your answer to four decimal places.



Solution:

First, find the measure for b.

$$c^2 = a^2 + b^2$$
$$13^2 = 12^2 + b^2$$
$$169 = 144 + b^2$$
$$b^2 = 169 - 144$$
$$b^2 = 25$$
$$b = 5$$

The ratios for $\angle A$ are as follows:

$$\sin A = \frac{12}{13} \doteq 0.923\,076\,923 \doteq 0.9231$$

$$\cos A = \frac{5}{13} \doteq 0.384\,615\,384 \doteq 0.3846$$

$$\tan A = \frac{12}{5} = 2.4 = 2.4000$$

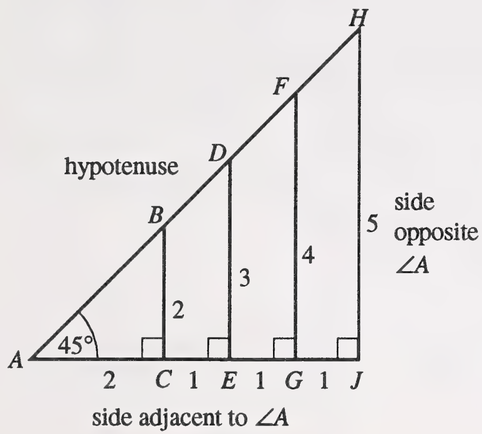
The ratios for $\angle B$ are as follows:

$$\sin B = \frac{5}{13} \doteq 0.384\,615\,384 \doteq 0.3846$$

$$\cos B = \frac{12}{13} \doteq 0.923\,076\,923 \doteq 0.9231$$

$$\tan A = \frac{5}{12} \doteq 0.416\,666\,666 \doteq 0.4167$$

It is important to know that the primary trigonometric ratios for any particular angle are always the same no matter what the lengths of the sides are. Study the following sketch. Notice that all the triangles are similar since $\angle A$ is common to all the triangles and $\angle C$, $\angle E$, $\angle G$, and $\angle J$ are right angles. Remember that when two angles of one triangle are equal to two angles of another triangle, the third angle of both triangles must be equal as well.



The tangent ratio for the four right triangles would be $\tan A = \dfrac{\text{side opposite } \angle A}{\text{side adjacent to } \angle A}$.

In ΔABC, $\tan A = \frac{2}{2}$.

In ΔADE, $\tan A = \frac{3}{3}$.

In ΔAFG, $\tan A = \frac{4}{4}$.

In ΔAHJ, $\tan A = \frac{5}{5}$.

Thus, $\tan A = \frac{2}{2} = \frac{3}{3} = \frac{4}{4} = \frac{5}{5} = 1.0000$.

$$\begin{aligned} \therefore \tan A &= \tan 45^\circ \\ &= 1.0000 \end{aligned}$$

In each similar triangle, this trigonometric ratio is the same. The same procedure can be used to show that the other trigonometric ratios are the same in the four similar triangles.

It is time to do some more problems on your own. For more similar applications, you may wish to do the **Extra Help** section. For more challenging situations you may do the **Extensions** section.

Recall: In all triangles, the three angles equal to 180°.

$$\begin{aligned} AE &= 2+1 \\ &= 3 \end{aligned}$$

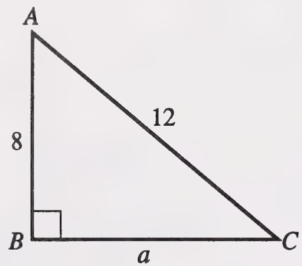
$$\begin{aligned} AG &= 2+1+1 \\ &= 4 \end{aligned}$$

$$\begin{aligned} AJ &= 2+1+1+1 \\ &= 5 \end{aligned}$$

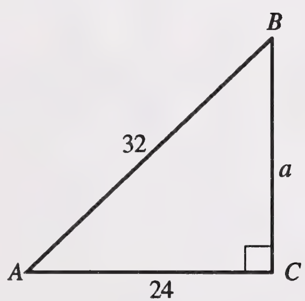
Do at least one problem from each of questions 1, 2, 3, and 4.

1. In each triangle, find the value of cos A. Leave your answers in fraction form. Simplify the fractions where possible.

a.

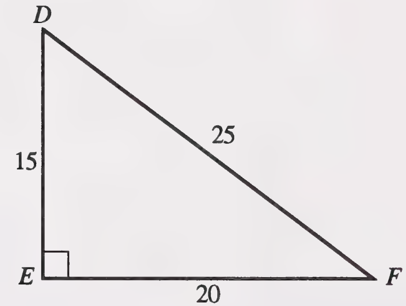


b.

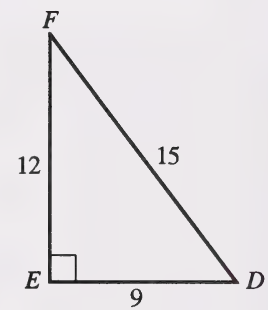


2. In ΔDEF, find the sine and cosine values for $\angle D$ and $\angle F$. Simplify the fractions where possible.

a.

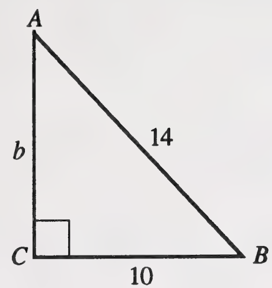


b.

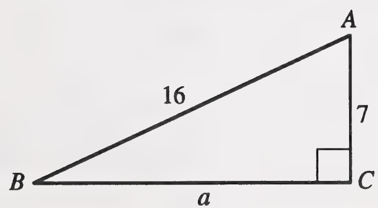


3. For each triangle, find the specified ratios. Round your answer to four decimal places.

   a. Find sin A.

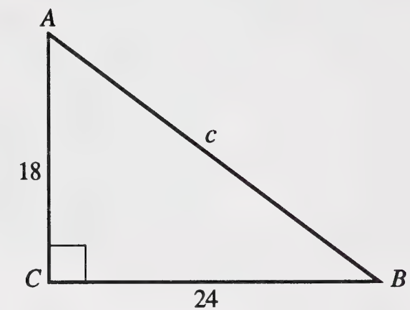


   b. Find cos A.

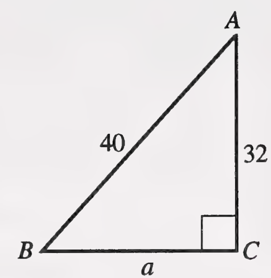


4. Find the primary trigonometric ratios for each acute angle in the given triangles. Provide two answers for each: one as a fraction in simplest form and the other as a decimal rounded to four places.

   a.



   b.



For solutions to **Activity 4**, turn to the **Appendix**, **Topic 3**.

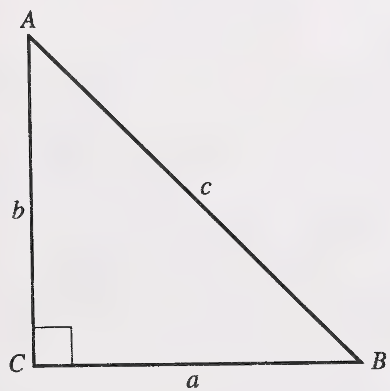
If you require help, do the Extra Help section.

If you want more challenging explorations, do the Extensions section.

You may decide to do both.

Extra Help

You have seen that labelling right-angled triangles properly is very important. It is equally important to correctly identify the opposite side and the adjacent side of any chosen reference angle. Review the labelling of right triangles in the example that follows.



The same pattern exists if other names for the vertices are used.

The next example shows how a simple sketch can be used to identify the opposite side and the adjacent side for the two acute angles in a right triangle.

Recall that acute angles are angles that measure less than 90°.

How many acute angles can you find in the following construction scene.

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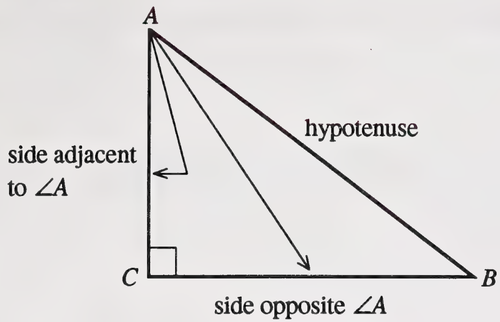
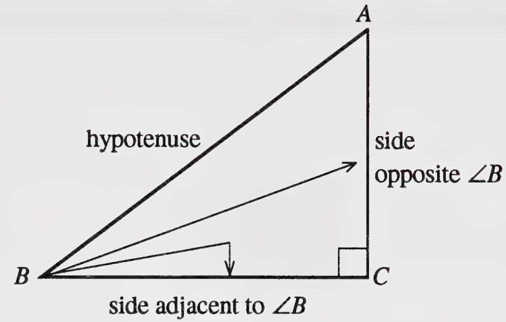




To easily recall the three primary trigonometric ratios, the following association technique can be used. Such a trick is called an acronym.

$$\sin\ \theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\ \theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan\ \theta = \frac{\text{opposite}}{\text{adjacent}}$$

SOH	CAH	TOA

A phrase can be used to help remember this letter arrangement. A common one is as follows:

Soak	A	Toe
SOH	CAH	TOA

You probably could coin other phrases to help remember this association. You can use whatever works best for you.

Recall: The hypotenuse is the longest side of a right triangle. It is the side opposite the right angle.

Note:

- A right-angled triangle can be abbreviated as $rt\Delta$.
- A right angle can be abbreviated as $rt\angle$.

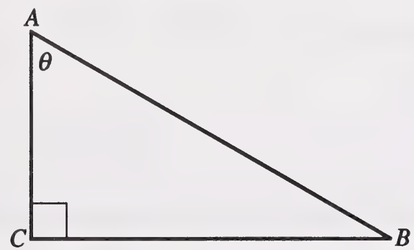
Sine
Opposite
Hypotenuse

Cosine
Adjacent
Hypotenuse

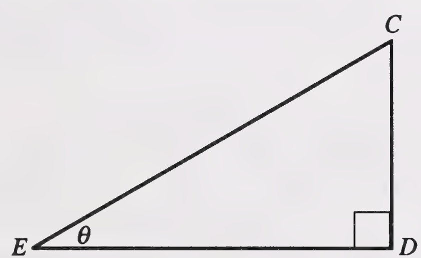
Tangent
Opposite
Adjacent

To develop a better understanding of right triangles and the trigonometric ratios, do all the following practice questions. If you have made errors, review the material again.

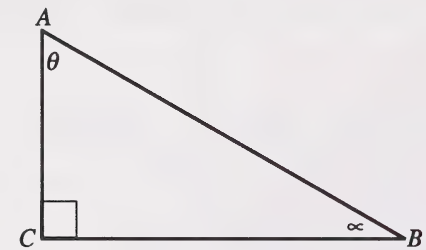
1. On the triangle that follows, identify the hypotenuse, the side opposite the reference angle, and the side adjacent to the reference angle. The reference angle is labelled θ.



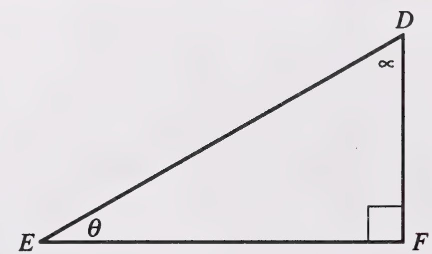
2. On the triangle that follows, identify the hypotenuse, the side opposite the reference angle, and the side adjacent to the reference angle. The reference angle is labelled θ.



3. Use small letters to name the primary trigonometric ratios for θ and $\propto$ in ΔABC.



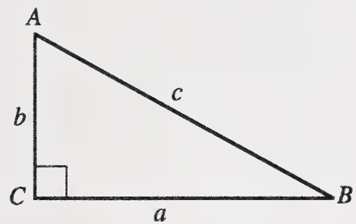
4. Use small letters to name the primary trigonometric ratios for θ and $\propto$ in ΔDEF.



For solutions to **Extra Help**, turn to the **Appendix**, **Topic 3**.

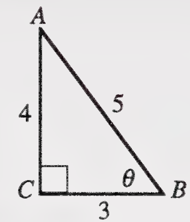
Extensions

Earlier, in this unit you used small letters to name the sides of a right triangle. For example, in ΔABC, you used a, b, and c.



Then, you used values for a and b which, when squared and added, resulted in a sum that was a perfect square. For example, in ΔABC $a = 3$ and $b = 4$. When 3 and 4 are squared and the squares added, you get $3^2 + 4^2 = 9 + 16 = 25$. The sum 25 is a perfect square. Side c is the square root of 25, which is 5. All the values used are whole numbers. There is no reason why these values could not be fractions, decimal numerals, or radicals. The following examples show how the ratios are calculated when the values are whole numbers, fractions, decimals, or radicals. The ratios have been rounded to four decimal places where necessary.

Whole Numbers

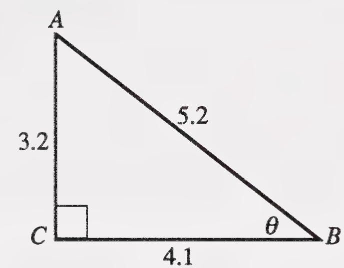


$\tan\ \theta = \frac{4}{3}$
$\doteq 1.3333$

$\sin\ \theta = \frac{4}{5}$
$= 0.8$

$\cos\ \theta = \frac{3}{5}$
$= 0.6$

Decimal Numerals

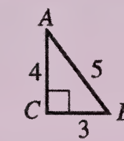


$\tan\ \theta = \frac{3.2}{4.1}$
$\doteq 0.7805$

$\sin\ \theta = \frac{3.2}{5.2}$
$\doteq 0.6154$

$\cos\ \theta = \frac{4.1}{5.2}$
$\doteq 0.7885$

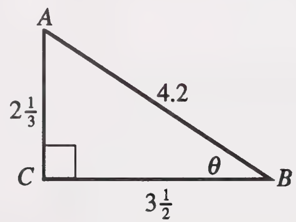
Recall:



LS	RS
c^2	$a^2 + b^2$
5^2	$3^2 + 4^2$
25	$9 + 16$
25	25

LS = RS

Fractions and Decimal Numerals

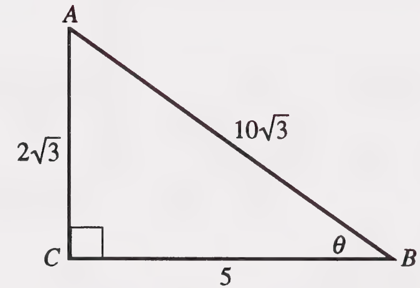


$$\tan\theta = \frac{2\frac{1}{3}}{3\frac{1}{2}} = \frac{\frac{7}{3}}{\frac{7}{2}} = \frac{2}{3} \doteq 0.6667$$

$$\sin\theta = \frac{2\frac{1}{3}}{4.2} = \frac{\frac{7}{3}}{4.2} = \frac{5}{9} \doteq 0.5556$$

$$\cos\theta = \frac{3\frac{1}{2}}{4.2} = \frac{\frac{7}{2}}{4.2} = \frac{5}{6} \doteq 0.8333$$

Radicals



$$\tan\theta = \frac{2\sqrt{3}}{5} \doteq 0.6928$$

$$\sin\theta = \frac{2\sqrt{3}}{10\sqrt{3}} = \frac{2}{10} = \frac{1}{5} = 0.2$$

$$\cos\theta = \frac{5}{10\sqrt{3}} = \frac{1}{2\sqrt{3}} \doteq 0.2887$$

$$\frac{\frac{7}{3}}{4.2} = \frac{\frac{7}{3}}{\frac{42}{10}} = \frac{7}{3} \div \frac{42}{10} = \frac{\overset{1}{\cancel{7}}}{3} \times \frac{10}{\underset{6}{\cancel{42}}} = \frac{10}{18} = \frac{5}{9}$$

Recall:

- $2\sqrt{3} = \sqrt{4 \times 3} = \sqrt{12}$
- $2\sqrt{3} \times 5 = 10\sqrt{3}$
- In $\frac{5}{10\sqrt{3}}$, the 5 cancels with the 10 to give $\frac{1}{2\sqrt{3}}$.
- $\frac{1}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{6}$ (This is simplifying a radical.)

If you wish, the trigonometric ratios here can be left in radical form. Such ratios would then be exact.

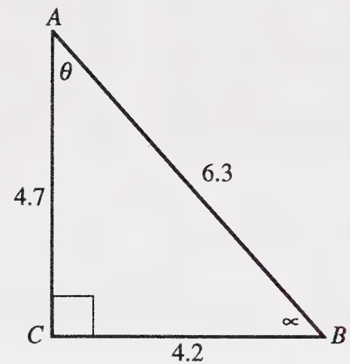
$$\tan\theta = \frac{2\sqrt{3}}{5}$$

$$\cos\theta = \frac{5}{10\sqrt{3}} = \frac{5}{10\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{5\sqrt{3}}{30} = \frac{\sqrt{3}}{6}$$

$$\sin\theta = \frac{2\sqrt{3}}{10\sqrt{3}} = \frac{2}{10} \times \frac{\cancel{\sqrt{3}}^{1}}{\cancel{\sqrt{3}}_{1}} = \frac{1}{5}$$

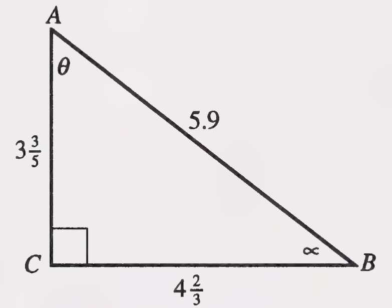
To apply these additional concepts, do the questions that follow. Use a calculator to find the answers.

1. Express the primary trigonometric ratios for θ and $\propto$ as decimal numerals. Round your answer to four places.

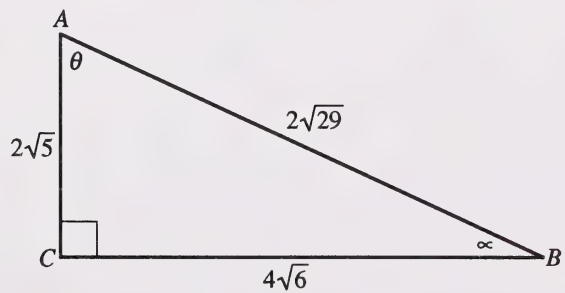


Note: A fraction that has a radical in the denominator is not in simplest form. To simplify, rationalize the denominator. Be sure to multiply both parts of the fraction by whatever radical needs to be changed to a rational number.

2. Express the primary trigonometric ratios for θ and $\propto$ as decimal numerals. Round your answer to four places.



3. Find the primary trigonometric ratios for θ and $\propto$. Express each ratio as a radical fraction in simplest form.



For solutions to **Extensions**, turn to the **Appendix**, **Topic 3**.

Topic 4 Applying Trigonometric Ratios

Introduction

Rheanne spent a week of her summer holdiays visiting her uncle who is a forest ranger. The two spent some time together at the top of a lookout tower. Her uncle selected several landmarks, at different distances from the lookout tower, and asked Rheanne to guess the distance to each landmark. Rheanne guessed and found out that her estimates were not even close. Her uncle then showed how accurate readings are made using trigonometry. In this topic you will also learn how this is done by indirect measurement.

What Lies Ahead

Throughout this topic you will learn to

1. apply trigonometric ratios to determine the measure of an angle of a given right triangle
2. apply trigonometric ratios to determine the length of a side of a given right triangle
3. apply trigonometric ratios to solve right triangles
4. apply trigonometric ratios to solve word problems involving an unknown side or angle of a right triangle

Now that you know what to expect, turn the page to begin your study of applying trigonometric ratios.

Exploring Topic 4

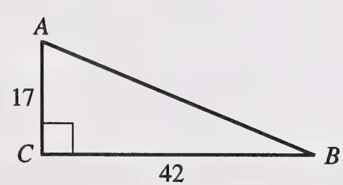
Activity 1

Apply trigonometric ratios to determine the measure of an angle of a given right triangle.

When the measure of two sides of a right triangle are given, and you are asked to find the measure of a particular angle, you must first determine which trig ratio to use. Then, change the fraction to a decimal numeral rounded to four decimal places. Study the following example.

Example 1

In following triangles, determine the measure of $\angle B$ to the nearest degree.

- 

Solution:

Determine how the given sides relate to the required angle.

$\overline{AC}$ is opposite $\angle B$. $\overline{CB}$ is adjacent to $\angle B$. Therefore, you will use $\tan B$.

$$\tan B = \frac{17}{42}$$

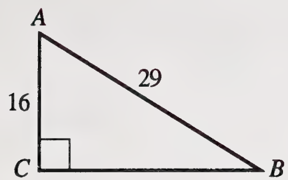
Now use your calculator.

Enter	Display
17	17
÷	17
42	42
=	0.404761904
INV	0.404761904
tan	22.03622694

Therefore, $\angle B \doteq 22°$.

Use the trigonometric ratio that contains the known sides.

$$\tan \theta = \frac{\text{side opposite}}{\text{side adjacent}}$$

• 

Solution:

To find $\angle B$, compare the side opposite $\angle B$ and the hypotenuse. Change the ratio to a decimal numeral rounded to four decimal places.

$$\sin B = \frac{16}{29}$$

Use a calculator.

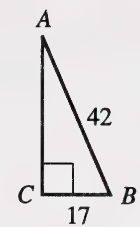
Enter	Display
16	16
$\boxed{\div}$	16
29	29
$\boxed{=}$	0.551724137
$\boxed{\text{INV}}$	0.551724137
$\boxed{\sin}$	33.48537662

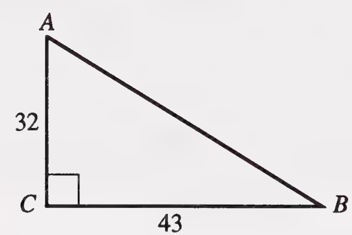
Therefore, $\angle B \doteq 33°$.

Apply this new concept in the following questions.

Do either the odd or the even problems. Use a calculator or a table to find the angle measure.

1. Find $\angle A$ to the nearest degree.

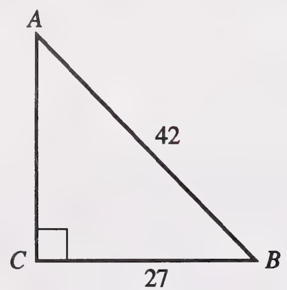
a. 

b. 

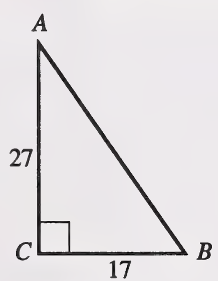
$$\sin \theta = \frac{\text{side opposite}}{\text{hypotenuse}}$$

2. Find $\angle B$ to the nearest degree.

a.



b.



For solutions to **Activity 1**, turn to the **Appendix**, **Topic 4**.

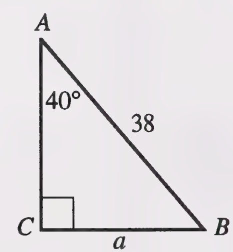
Activity 2

Apply trigonometric ratios to determine the length of a side of a given right triangle.

You can also find the length of a side of a triangle if the measures of at least one side and one acute angle are known. Study the example given.

Example 2

Solve for a. Round to the nearest whole number.



Solution:

$\overline{CB}$ is opposite the given angle (40°). $\overline{AB}$, the given side, is the hypotenuse. Therefore, use $\sin A$.

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$

Rewrite the equation, and solve for the unknown quantity.

$$\sin 40° = \frac{a}{38} \quad \text{(Multiply both side by 38.)}$$
$$38(\sin 40°) = a$$
$$a = 38(\sin 40°)$$

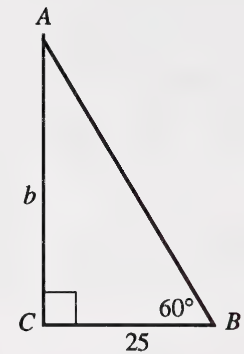
Use your calculator.

Enter	Display
40	40
$\boxed{\text{sin}}$	0.642787609
$\boxed{\times}$	0.642787609
38	38
$\boxed{=}$	24.42592917

Therefore, $a \doteq 24$ units.

Example 3

Solve for b. Round to the nearest whole number.

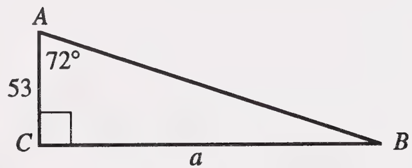


Solution:

$$\tan B = \frac{b}{25}$$
$$\tan 60° = \frac{b}{25}$$
$$25(\tan 60°) = b$$
$$b = 25(\tan 60°)$$
$$\doteq 25(1.732\,050\,808)$$
$$\doteq 43.301\,270\,19$$
$$\doteq 43 \text{ units}$$

Example 4

Solve for a. Round to the nearest whole number.



Solution:

$$\tan A = \frac{a}{53}$$

$$\tan 72° = \frac{a}{53}$$

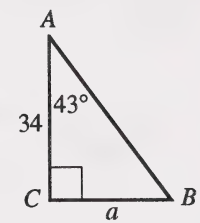
$$53(\tan 72°) = a$$

$$\begin{aligned} a &= 53(\tan 72°) \\ &\doteq 53(3.077\,683\,537) \\ &\doteq 163.117\,2275 \\ &\doteq 163 \text{ units} \end{aligned}$$

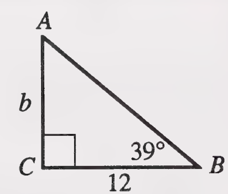
Now apply the concepts needed to find the measure of a particular side of a right triangle when the measures of one angle and one side are given.

Do all of the following problems.

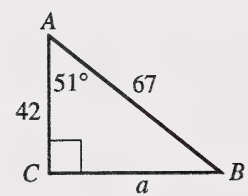
1. Find the measure of a. Round to the nearest whole number.



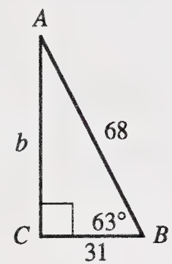
2. Find the measure of b. Round to the nearest whole number.



3. Find the measure of a. Round to the nearest whole number. Check your answer by using a different ratio.



4. Find the measure of b. Round to the nearest whole number. Check your answer by using a different ratio.



For solutions to **Activity 2**, turn to the **Appendix**, **Topic 4**.

Activity 3

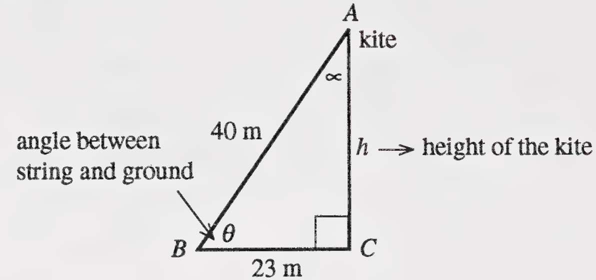
Apply trigonometric ratios to solve right triangles.

Example 5

One Saturday morning Stephan decided to try out his home-made kite. He attached a 40 m string to the kite, and tied it down. He then paced 23 m to a point where he thought the kite would be directly overhead. Sketch a simple diagram. Round your answers to the nearest whole number.

- What angle did the string form with the ground?

Solution:



Find the measure of θ.

$$\cos\ \theta = \frac{23}{40}$$
$$= 0.5750$$

Use a calculator.

Enter	Display
23	23
÷	23
40	40
=	0.575
INV	0.575
cos	54.90036781

Therefore, $\theta \doteq 55°$.

- What angle did the string form with the kite?

Solution:

Find the measure of $\propto$.

$$\sin \propto = \frac{23}{40}$$
$$= 0.5750$$

Use a calculator.

Enter	Display
23	23
$\boxed{+}$	23
40	40
$\boxed{=}$	0.575
$\boxed{\text{INV}}$	0.575
$\boxed{\sin}$	35.0996322

Therefore, $\propto \doteq 35°$.

Once θ is found, $\propto$ can also be found as follows:

$$\propto + \theta = 90°$$
$$\propto \doteq 90° - 55°$$
$$\doteq 35°$$

(If one angle in a triangle is 90°, then the sum of the other two angles is 90°.)

- How high his kite is when the string is fully extended?

Solution:

To find h one of five methods can be used.

Method 1

$$\tan \theta = \frac{h}{23}$$
$$\tan 55° \doteq \frac{h}{23}$$
$$h \doteq 23(\tan 55°)$$
$$\doteq 23 \times 1.428\,148\,007$$
$$\doteq 32.847\,404\,15$$
$$\doteq 33 \text{ m}$$

Method 2

$$\sin \theta = \frac{h}{40}$$
$$\sin 55° \doteq \frac{h}{40}$$
$$h \doteq 40(\sin 55°)$$
$$\doteq 40 \times 0.819\,152\,044$$
$$\doteq 32.766\,081\,77$$
$$\doteq 33 \text{ m}$$

Method 3

$$\tan \propto = \frac{23}{h}$$

$$\tan 35° \doteq \frac{23}{h}$$

$$h \doteq \frac{23}{\tan 35°}$$

$$\doteq \frac{23}{0.700\,207\,538}$$

$$\doteq 32.847\,404\,16$$

$$\doteq 33 \text{ m}$$

Method 4

$$\cos \propto = \frac{h}{40}$$

$$\cos 35° \doteq \frac{h}{40}$$

$$h \doteq 40(\cos 35°)$$

$$\doteq 40 \times 0.819\,152\,044$$

$$\doteq 32.766\,081\,77$$

$$\doteq 33 \text{ m}$$

Method 5

$$40^2 = 23^2 + h^2$$

$$1600 = 529 + h^2$$

$$h^2 = 1600 - 529$$

$$h^2 = 1071$$

$$\sqrt{h^2} = \sqrt{1071}$$

$$h \doteq 32.726\,136\,34$$

$$\doteq 33 \text{ m}$$

The height of the kite is approximately 33 m.

The summary of triangle *ABC* is as follows:

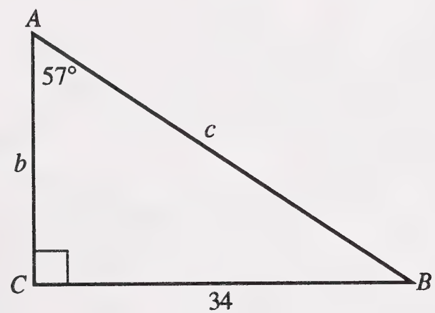
$\angle A$ or $\propto \doteq 35°$	$a = 23$ m
$\angle B$ or $\theta \doteq 55°$	$c = 40$ m
$\angle C = 90°$	b or $h \doteq 33$ m

No matter which method is chosen, the final answer is the same. There is no set rule specifying which one should be used. Finding all the measures of the missing angles and missing sides of a right triangle is called **solving a right triangle**.

Study the following example.

Example 6

Solve the following triangle. Measure each angle to the nearest degree, and measure each side to the nearest whole number.



Solution:

To solve triangle ABC, we need to find the measure of b, c, and $\angle B$.

Find the measure of b.

$$\begin{aligned} \tan 57° &= \frac{34}{b} \\ b &= \frac{34}{\tan 57°} \\ &\doteq \frac{34}{1.539\,864\,964} \\ &\doteq 22.079\,858\,17 \\ &\doteq 22 \text{ units} \end{aligned}$$

Find the measure of c.

$$\begin{aligned} \sin 57° &= \frac{34}{c} \\ c &= \frac{34}{\sin 57°} \\ &\doteq \frac{34}{0.838\,670\,568} \\ &\doteq 40.540\,351\,95 \\ &\doteq 41 \text{ units} \end{aligned}$$

Find the measure of $\angle B$.

$$\begin{aligned} \angle B &= 180° - (90 + 57)° \\ &= 180° - 147° \\ &= 33° \end{aligned}$$

Angle B can also be found in this way.

$$\sin B \doteq \frac{22}{41} \quad \text{or} \quad \cos B \doteq \frac{34}{41} \quad \text{or} \quad \tan B \doteq \frac{22}{34}$$

However, because the values of b and c are rounded, the value of $\angle B$ will not be as accurate. As a general rule use the given values where possible to find the missing angles or sides.

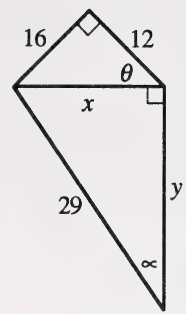
The summary of triangle ABC is as follows:

$\angle A = 57°$ $a = 34$ units
$\angle B = 33°$ $b \doteq 22$ units
$\angle C = 90°$ $c \doteq 41$ units

Sometimes you may have to solve triangles which are combined together. The same trigonometric ratios that are used to solve one triangle may be used to solve combined triangles.

Example 7

Solve for each unknown. Round your answer to two decimal places if necessary.



Solution:

First solve for x.

$$c^2 = a^2 + b^2$$
$$x^2 = 12^2 + 16^2$$
$$x^2 = 144 + 256$$
$$x^2 = 400$$
$$\sqrt{x^2} = \sqrt{400}$$
$$x = 20 \text{ units}$$

Solve for θ.

$$\tan\ \theta = \frac{16}{12}$$
$$\tan\ \theta \doteq 1.333\,333\,333$$
$$\theta \doteq 53.130\,102\,35$$
$$\doteq 53.13°$$

Solve for y.

$$c^2 = a^2 + b^2$$
$$29^2 = y^2 + 20^2$$
$$841 = y^2 + 400$$
$$y^2 = 841 - 400$$
$$y^2 = 441$$
$$\sqrt{y^2} = \sqrt{441}$$
$$y = 21 \text{ units}$$

Solve for $\propto$.

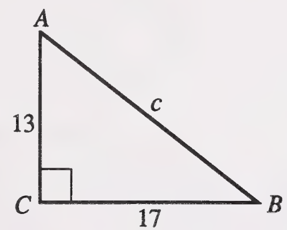
$$\cos\ \propto = \frac{y}{29}$$
$$\cos\ \propto = \frac{21}{29}$$
$$\cos\ \propto \doteq 0.724\,137\,931$$
$$\propto \doteq 43.602\,818\,97$$
$$\doteq 43.60°$$

Now it is time to do some questions on your own.

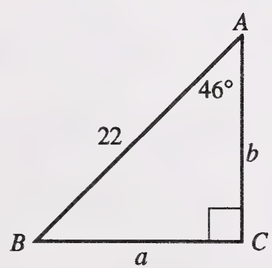
Do question 1 or 2 and question 3.

1. Using a calculator solve the triangles provided. Measure each angle to the nearest degree, and measure each side to the nearest whole number.

a.

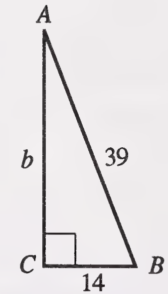


b.

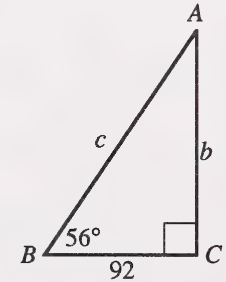


2. Solve the following triangles. Measure each angle to the nearest degree, and measure each side to the nearest whole number.

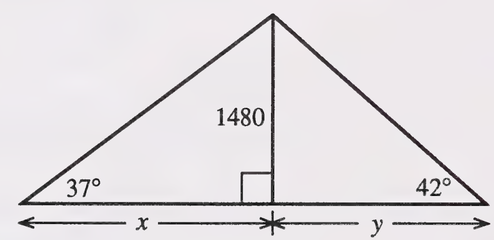
a.



b.



3. Find each unknown indicated.



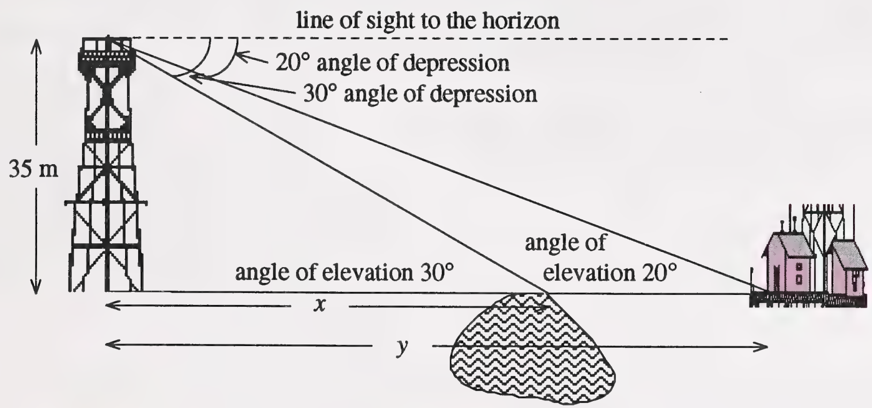
For solutions to **Activity 3**, turn to the **Appendix**, **Topic 4**.

Activity 4

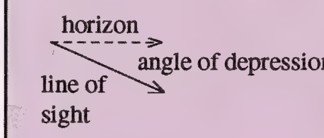
Apply trigonometric ratios to solve word problems involving an unknown side or angle of a right triangle.

Go back to the situation described in the introduction to this topic. Sketch a diagram to show what information is provided and what needs to be found. The landmarks are a small lake and the buildings of a coal mine. The height of the tower is 35 m.

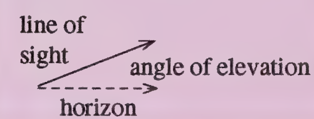
Using a special instrument called a transit, Rheanne's uncle found two angles of depression: one for the lake and the other for the buildings of a coal mine. The angle for the lake was 30°, and the angle for the buildings was 20°. Her uncle also explained that if an angle reading was to be taken at the lake and at the coal mine, the angles of elevation would also be 30° and 20° respectively.



When looking down from a line to the horizon, an angle of depression is formed.



When looking up from the horizon, an angle of elevation is formed.



From the sketch you can see that there are two pairs of angles. Each pair is formed when a transversal intersects two parallel lines. Such angles are called alternate angles, and they are equal in measure. This means that the angle of depression is equal to its corresponding angle of elevation.

Rheanne was shown that the distances to the lake and to the buildings of the coal mine can be calculated by using the tangent ratio, and that these distances would be quite accurate even when rounded to the nearest metre.

Solve for x (the distance to the lake).

$$\tan 30° = \frac{35}{x}$$

$$x = \frac{35}{\tan 30°}$$

$$\doteq \frac{35}{0.577\,350\,269}$$

$$\doteq 60.621\,778\,27$$

$$\doteq 61 \text{ m}$$

The lake is about 61 m from the tower.

Solve for y (the distance to the buildings of a coal mine).

$$\tan 20° = \frac{35}{y}$$

$$y = \frac{35}{\tan 20°}$$

$$\doteq \frac{35}{0.363\,970\,234}$$

$$\doteq 96.161\,709\,68$$

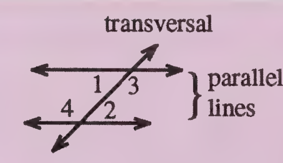
$$\doteq 96 \text{ m}$$

The buildings of the coal mine are about 96 m from the tower.

This is only one example of many situations where indirect measurement is the easiest way to calculate distances.

It is very important to know the primary trigonometric ratios and the relationship between angles of depression and angles of elevation.

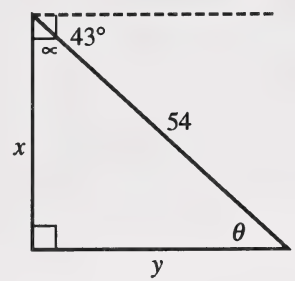
Using all the concepts mentioned in this topic, try the following problems on your own.



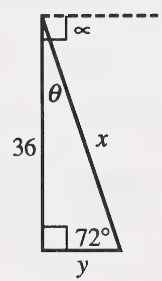
$\angle 1 = \angle 2$ (alternate angles)
$\angle 3 = \angle 4$ (alternate angles)

Do either the odd- or even-numbered problems.

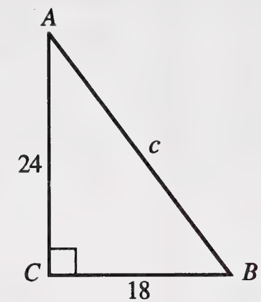
1. Find the measures of θ and $\propto$ to the nearest degree, and find the measures of x and y to the nearest tenth (one decimal place).



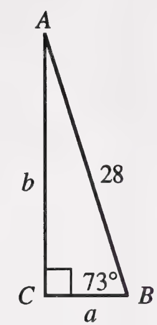
2. Find the measures of θ and $\propto$ to the nearest degree, and find the measures of x and y to the nearest tenth (one decimal place).



3. Solve ΔABC. Measure each angle to the nearest degree, and measure each side to the nearest tenth.



4. Solve ΔABC. Measure each angle to the nearest degree, and measure each side to the nearest tenth.



5. To be absolutely on the safe side, a ladder should make an angle of 74° with the ground. If a ladder is 7 m long, how far will it reach up the wall? Round your answer to one decimal place.

6. A tree casts a shadow 37 m long when the sun's rays are at an angle of 28° to the ground. What is the height of the tree? Round your answer to one decimal place.

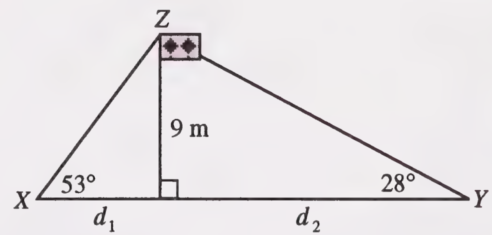
7. A train track rises 75 m over a horizontal distance of 1.5 km. What is the angle of elevation of the track to the nearest degree?

8. The angle of elevation of a road is 5°. How much does the road rise over a direct distance of 230 m measured along the road? Round your answer to one decimal place.

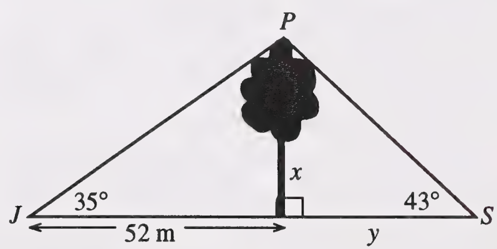
9. The angle of depression from an aircraft to its landing strip is 18°. If the altitude of the aircraft is 1690 m, what is the horizontal distance from the plane to the landing strip? Round your answer to the nearest metre.

10. From the top of a cliff 105 m above a river, an angle reading is taken to a point directly across the river. This angle measures 42° and is the angle of depression. Find the width of the river to the nearest metre.

11. The angles of elevation to the top of a flag pole from points on either side of it are as shown. If the flag pole is 9 m tall, what is the distance from X to Y? Round your answer to the nearest metre.



12. Jackie (J) and Saundra (S) stand on opposite sides of a tree at point J and S. Jackie is 52 m from the tree. What is the distance between Jackie and Saundra to the nearest metre ?

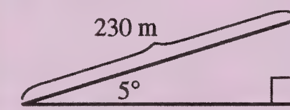


For solutions to **Activity 4**, turn to the **Appendix**, **Topic 4**.

Begin each solution with a simple diagram showing what is given and what is required.

You will need to first solve each triangle separately.

distance measured along the road



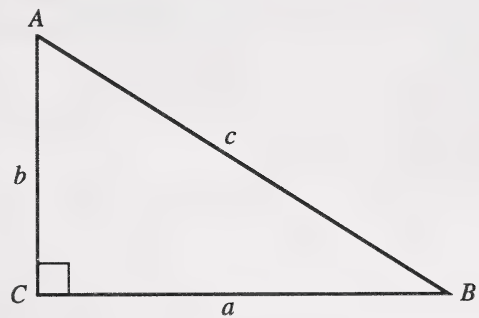
If you require help, do the Extra Help section.

If you want more challenging explorations, do the Extensions section.

You may decide to do both.

Extra Help

Before you do the problems, remember these points. Use the following triangle as a reference.



- In ΔABC the side opposite $\angle A$ is a, and the side adjacent to $\angle A$ is b. The side opposite $\angle B$ is b, and the side adjacent to $\angle B$ is a. The hypotenuse is c.

- $\angle A$ and $\angle B$ are the two acute angles of right triangle ΔABC.

- $\sin A = \dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{a}{c}$ $\qquad \cos A = \dfrac{\text{adjacent}}{\text{hypotenuse}} = \dfrac{b}{c}$ $\qquad \tan A = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{a}{b}$

- $\sin B = \dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{b}{c}$ $\qquad \cos B = \dfrac{\text{adjacent}}{\text{hypotenuse}} = \dfrac{a}{c}$ $\qquad \tan B = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{b}{a}$

Recall the following:

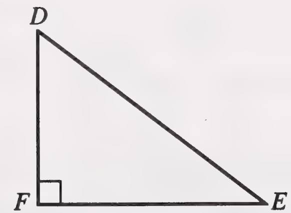
S → sin
O → opposite
H → hypotenuse

C → cos
A → adjacent
H → hypotenuse

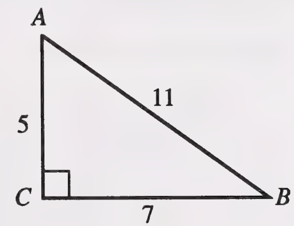
T → tan
O → opposite
A → adjacent

Use the previous information to answer the following questions.

1. Give the three primary trigonometric ratios for each acute angle in ΔDEF.



2. Use the following triangle to answer the following questions.



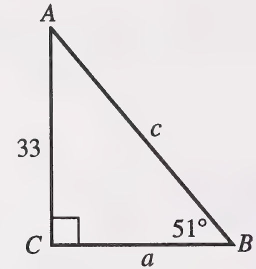
   a. Give the three primary trigonometric ratios for each acute angle in ΔABC in fraction form.

   b. Give the three primary trigonometric ratios for each acute angle in ΔABC in decimal form rounded to four places.

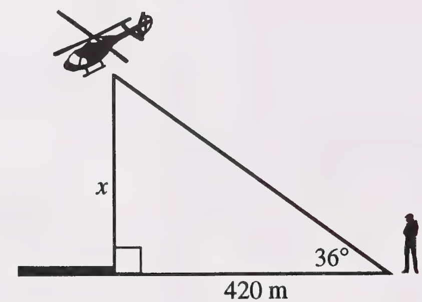
3. Use a calculator to find the measure of the angle that corresponds to each of the following. Round your answers to the nearest degree.

   a. $\cos A = 0.7042$
   b. $\sin A = 0.2009$
   c. $\tan A = 1.7634$
   d. $\tan B = 0.5036$
   e. $\sin B = 0.8961$
   f. $\cos B = 0.0337$

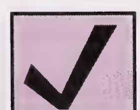
4. Solve ΔABC. Round all answers to the nearest whole number.



5. Use the given sketch. How high is the helicopter above the landing pad? Round your answer to the nearest metre.



6. Find the angle of elevation to the top of a tower 23 m high from a point 16 m from its base. Round your answer to the nearest degree.

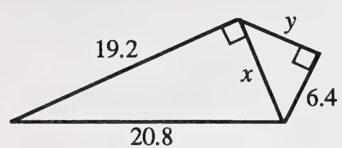


For solutions to **Extra Help**, turn to the **Appendix, Topic 4**.

Extensions

In many situations where trigonometry is involved, several steps must be used to find the final solution to problems. Keep this in mind as you try the following problems.

1. What is the combined area for the two triangles shown? Round the answer to one decimal place if necessary.



2. The distance between two buildings is 150 m. From the path, which is equidistant from each building, the angles of elevation of their tops are 16° and 22° rescpectively. How much taller is one building than the other? Round your answer to one decimal place where needed.

3. From a point 150 m from the base of a building, the angle of elevation to the top of the building is 46°. It was measured with a transit 1.5 m tall. If the angle of elevation to the top of a flagpole on top of the building is 48°, find the height of the flagpole itself. What is the combined height of the flagpole and the building? Round your answers to the nearest tenth of a metre.

4. From the top of a 45 m cliff, two boats in line are observed with angles of depression of 12° and 18° respectively. How far apart are the two boats? Round all values used to the nearest tenth.

For solutions to **Extensions**, turn to the **Appendix**, **Topic 4**.

Recall: The area for any triangle can be found using $A = \frac{1}{2}bh$, where b is the base and h is the perpendicular height.

Unit Summary

What You Have Learned

In this unit you have learned the following:

- the theorem of Pythagoras and how to use it to find the length of a side of a right triangle
- similar triangles and their properties
- using the properties of similar triangles to develop the tangent, sine, and cosine ratios
- $\tan = \dfrac{\text{side opposite}}{\text{side adjacent}}$
- $\sin = \dfrac{\text{side opposite}}{\text{hypotenuse}}$
- $\cos = \dfrac{\text{side adjacent}}{\text{hypotenuse}}$
- using a calculator to find the ratio given the measure of an angle and to find the measure of an angle given a particular case
- finding the measures of the acute angles in a right triangle given measures of any two sides
- finding the measures of an acute angle in a right triangle given one of its trigonometric ratios
- applying trigonometric ratios to determine the measure of an angle, or length of a side in a given right triangle
- applying trigonometric ratios to solve right triangles
- applying trigonometric ratios to solve word problems involving an unknown side or angle of a right triangle

You are now ready to

complete the **Unit Assignment**.

Appendix

Solutions

Review

Topic 1 The Theorem of Pythagoras

Topic 2 Properties of Similar Triangles

Topic 3 Developing and Finding Trigonometric Ratios

Topic 4 Applying Trigonometric Ratios

Review

1. a. $\overline{QR}$ or p

   Recall: $\overline{QR}$ means line segment QR. The bar above the letters are needed if using only symbols.

   b. $\angle P$ or $\angle QPR$

   c. $\angle Q$ or $\angle PQR$, $\angle R$ or $\angle PRQ$

   d. $\overline{PR}$ or q, $\overline{QP}$ or r, $\overline{QR}$ or p

2. a. $\sqrt{49} = \sqrt{7 \times 7}$
   $= \sqrt{7^2}$
   $= 7$

Enter	Display
49	49
$\boxed{\sqrt{\ }}$	7

   b. $\sqrt{81} = \sqrt{9 \times 9}$
   $= \sqrt{9^2}$
   $= 9$

   c. $\sqrt{225} = \sqrt{15 \times 15}$
   $= \sqrt{15^2}$
   $= 15$

3. a. $16^2 = 16 \times 16$
   $= 256$

Enter	Display
16	16
$\boxed{x^2}$	256

   b. $25^2 = 25 \times 25$
   $= 625$

   c. $64^2 = 64 \times 64$
   $= 4096$

4. a. $\frac{5}{6} = \frac{x}{18}$
   $6x = 90$
   $\frac{6x}{6} = \frac{90}{6}$
   $x = 15$

   b. $\frac{x}{12} = \frac{2}{3}$
   $3x = 24$
   $\frac{3x}{3} = \frac{24}{3}$
   $x = 8$

   c. $\frac{5}{x} = \frac{15}{27}$
   $15x = 135$
   $\frac{15x}{15} = \frac{135}{15}$
   $x = 9$

   d. $\frac{14}{15} = \frac{56}{x}$
   $14x = 840$
   $\frac{14x}{4} = \frac{840}{14}$
   $x = 60$

e. $\frac{0.5}{6} = \frac{x}{0.2}$

$$6x = 0.1$$
$$\frac{6x}{6} = \frac{0.1}{6}$$
$$x = 0.01\overline{6}$$
$$\doteq 0.017$$

Recall: $0.01\overline{6} = 0.01666\ldots$
The six repeats itself forever.

f. $\frac{\frac{5}{6}}{3} = \frac{12}{x}$

$$\frac{5}{6}x = 36$$
$$\left(\frac{5}{6} \div \frac{5}{6}\right)x = 36 \div \frac{5}{6}$$
$$x = 36 \times \frac{6}{5}$$
$$= \frac{216}{5}$$
$$= 43.2$$

g. $\frac{7}{10} = \frac{x}{3\frac{3}{4}}$

$$10x = 3\frac{3}{4} \times 7$$
$$10x = \frac{15}{4} \times 7$$
$$10x = \frac{105}{4}$$
$$40x = 105$$
$$x = \frac{105}{40}$$
$$= 2\frac{25}{40}$$
$$= 2\frac{5}{8} \text{ or } 2.625$$

h. $\frac{3}{4.5} = \frac{14}{x}$

$$3x = 63$$
$$\frac{3x}{3} = \frac{63}{3}$$
$$x = 21$$

i. $\frac{x}{1\frac{3}{4}} = \frac{20}{1.5}$

$$\frac{x}{1.75} = \frac{20}{1.5}$$
$$1.5x = 35$$
$$\frac{1.5x}{1.5} = \frac{35}{1.5}$$
$$x = 23.\overline{3}$$

or $\frac{x}{1\frac{3}{4}} = \frac{20}{1.5}$

$$\frac{x}{1\frac{3}{4}} = \frac{20}{1\frac{1}{2}}$$
$$1\frac{1}{2}x = 1\frac{3}{4} \times 20$$
$$\frac{3}{2}x = 35$$
$$x = 35 \div \frac{3}{2}$$
$$= 35 \times \frac{2}{3}$$
$$= \frac{70}{3}$$
$$= 23\frac{1}{3}$$

Change $1\frac{3}{4}$ to 1.75 or change 1.5 to $1\frac{1}{2}$.

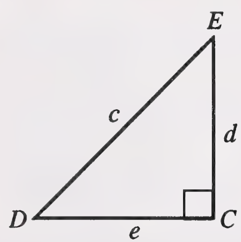
When a mixture of decimals and fractions is used, a change is made to get the same class of numbers before the solution is attempted.

Exploring Topic 1

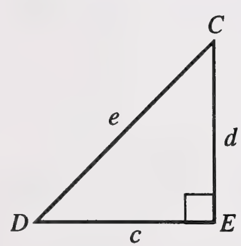
Activity 1

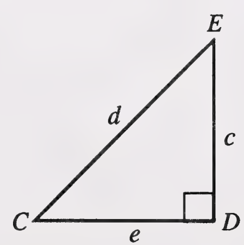
Draw and label the sides and angles of a triangle.

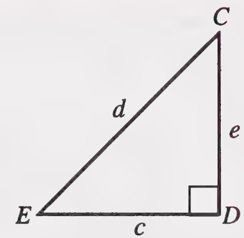
1. a.



Other labelling patterns can be used as long as C, D, and E represent the vertices.







There are two other possible ways to label the triangle.

b. $a = 4$
$b = 3$
$c = 5$

Activity 2

Find the length of a side of a given triangle, and solve problems by applying the Pythagorean theorem.

1. a.

$$x^2 = 12^2 + 5^2$$
$$x^2 = 144 + 25$$
$$x^2 = 169$$
$$\sqrt{x^2} = \sqrt{169}$$
$$x = 13 \text{ units}$$

b.

$$x^2 = 15^2 + 8^2$$
$$x^2 = 225 + 64$$
$$x^2 = 289$$
$$\sqrt{x^2} = \sqrt{289}$$
$$x = 17 \text{ units}$$

c. $$\begin{aligned} 5^2 &= x^2 + 3^2 \\ 25 &= x^2 + 9 \\ x^2 &= 25 - 9 \\ x^2 &= 16 \\ \sqrt{x^2} &= \sqrt{16} \\ x &= 4 \text{ units} \end{aligned}$$

d. $$\begin{aligned} 15^2 &= x^2 + 12^2 \\ 225 &= x^2 + 144 \\ x^2 &= 225 - 144 \\ x^2 &= 81 \\ \sqrt{x^2} &= \sqrt{81} \\ x &= 9 \text{ units} \end{aligned}$$

e. $$\begin{aligned} 20^2 &= 12 + x^2 \\ 400 &= 144 + x^2 \\ x^2 &= 400 - 144 \\ x^2 &= 256 \\ \sqrt{x^2} &= \sqrt{256} \\ x &= 16 \text{ units} \end{aligned}$$

f. $$\begin{aligned} 25^2 &= 20^2 + x^2 \\ 625 &= 400 + x^2 \\ x^2 &= 625 - 400 \\ x^2 &= 225 \\ \sqrt{x^2} &= \sqrt{225} \\ x &= 15 \text{ units} \end{aligned}$$

2. a. $$\begin{aligned} x^2 &= 5^2 + 8^2 \\ x^2 &= 25 + 64 \\ x^2 &= 89 \\ \sqrt{x^2} &= \sqrt{89} \\ x &\doteq 9.433\,981\,132 \\ &\doteq 9.4 \text{ units} \end{aligned}$$

b. $$\begin{aligned} x^2 &= 18^2 + 12^2 \\ x^2 &= 324 + 144 \\ x^2 &= 468 \\ \sqrt{x^2} &= \sqrt{468} \\ x &\doteq 21.633\,307\,65 \\ &\doteq 21.6 \text{ units} \end{aligned}$$

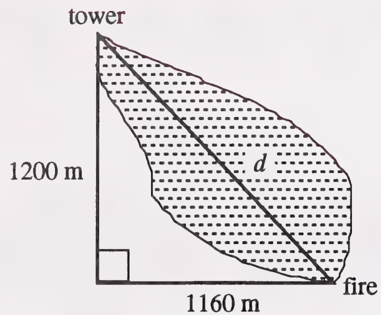
c. $$\begin{aligned} 22^2 &= x^2 + 9^2 \\ 484 &= x^2 + 81 \\ x^2 &= 484 - 81 \\ x^2 &= 403 \\ \sqrt{x^2} &= \sqrt{403} \\ x &\doteq 20.074\,8599 \\ &\doteq 20.1 \text{ units} \end{aligned}$$

d. $$\begin{aligned} 26^2 &= x^2 + 14^2 \\ 676 &= x^2 + 196 \\ x^2 &= 676 - 196 \\ x^2 &= 480 \\ \sqrt{x^2} &= \sqrt{480} \\ x &\doteq 21.908\,9023 \\ &\doteq 21.9 \text{ units} \end{aligned}$$

e. $$\begin{aligned} 33^2 &= 21^2 + x^2 \\ 1089 &= 441 + x^2 \\ x^2 &= 1089 - 441 \\ x^2 &= 648 \\ \sqrt{x^2} &= \sqrt{648} \\ x &\doteq 25.455\,844\,12 \\ &\doteq 25.5 \text{ units} \end{aligned}$$

f. $$\begin{aligned} 62^2 &= 41^2 + x^2 \\ 3844 &= 1681 + x^2 \\ x^2 &= 3844 - 1681 \\ x^2 &= 2163 \\ \sqrt{x^2} &= \sqrt{2163} \\ x &\doteq 46.508\,063\,82 \\ &\doteq 46.5 \text{ units} \end{aligned}$$

3.



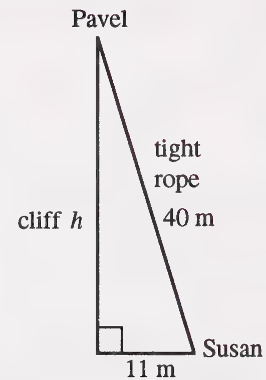
$$d^2 = (1200)^2 + (1160)^2$$
$$d^2 = 1\,440\,000 + 1\,345\,600$$
$$d^2 = 2\,785\,600$$
$$\sqrt{d^2} = \sqrt{2\,785\,600}$$
$$d \doteq 1669.011\,684$$
$$\doteq 1669 \text{ m}$$

The direct distance to the fire across the lake is about 1669 m.

$$\text{Land route} = 1200 + 1160$$
$$= 2360 \text{ m}$$

The direct route is $2360 \text{ m} - 1669 \text{ m} \doteq 691 \text{ m}$ shorter.

4.



$$40^2 = h^2 + 11^2$$
$$h^2 = 40^2 - 11^2$$
$$h^2 = 1600 - 121$$
$$\sqrt{h^2} = \sqrt{1479}$$
$$h \doteq 38.457\,769\,05$$
$$\doteq 38.5 \text{ m}$$

The height of the cliff is about 38. 5 m.

Extra Help

1. a.
$$\begin{aligned} x^2 &= 7^2 + 24^2 \\ x^2 &= 49 + 576 \\ x^2 &= 625 \\ \sqrt{x^2} &= \sqrt{625} \\ x &= 25 \text{ units} \end{aligned}$$

b.
$$\begin{aligned} x^2 &= 60^2 + 45^2 \\ x^2 &= 3600 + 2025 \\ x^2 &= 5625 \\ \sqrt{x^2} &= \sqrt{5625} \\ x &= 75 \text{ units} \end{aligned}$$

c.
$$\begin{aligned} 97^2 &= x^2 + 65^2 \\ 9409 &= x^2 + 4225 \\ x^2 &= 9409 - 4225 \\ x^2 &= 5184 \\ \sqrt{x^2} &= \sqrt{5184} \\ x &= 72 \text{ units} \end{aligned}$$

d.
$$\begin{aligned} 15^2 &= x^2 + 12^2 \\ 225 &= x^2 + 144 \\ x^2 &= 225 - 144 \\ x^2 &= 81 \\ \sqrt{x^2} &= \sqrt{81} \\ x &= 9 \text{ units} \end{aligned}$$

e.
$$\begin{aligned} 13^2 &= 12^2 + x^2 \\ 169 &= 144 + x^2 \\ x^2 &= 169 - 144 \\ x^2 &= 25 \\ \sqrt{x^2} &= \sqrt{25} \\ x &= 5 \text{ units} \end{aligned}$$

f.
$$\begin{aligned} 113^2 &= 15^2 + x^2 \\ 12\,769 &= 225 + x^2 \\ x^2 &= 12\,769 - 225 \\ x^2 &= 12\,544 \\ \sqrt{x^2} &= \sqrt{12\,544} \\ x &= 112 \text{ units} \end{aligned}$$

g.
$$\begin{aligned} x^2 &= 8^2 + 5^2 \\ x^2 &= 64 + 25 \\ x^2 &= 89 \\ \sqrt{x^2} &= \sqrt{89} \\ x &\doteq 9.433\,981\,132 \\ &\doteq 9.4 \text{ units} \end{aligned}$$

h.
$$\begin{aligned} x^2 &= 13^2 + 7^2 \\ x^2 &= 169 + 49 \\ x^2 &= 218 \\ \sqrt{x^2} &= \sqrt{218} \\ x &\doteq 14.764\,823\,06 \\ &\doteq 14.8 \text{ units} \end{aligned}$$

i. $16^2 = x^2 + 3^2$

$256 = x^2 + 9$

$x^2 = 256 - 9$

$x^2 = 247$

$\sqrt{x^2} = \sqrt{247}$

$x \doteq 15.716\,233\,65$

$\doteq 15.7$ units

j. $35^2 = x^2 + 16^2$

$1225 = x^2 + 256$

$x^2 = 1225 - 256$

$x^2 = 969$

$\sqrt{x^2} = \sqrt{969}$

$x \doteq 31.128\,764\,83$

$\doteq 31.1$ units

k. $27^2 = 11^2 + x^2$

$729 = 121 + x^2$

$x^2 = 729 - 121$

$x^2 = 608$

$\sqrt{x^2} = \sqrt{608}$

$x \doteq 24.657\,656\,01$

$\doteq 24.7$ units

l. $83^2 = 14^2 + x^2$

$6889 = 196 + x^2$

$x^2 = 6889 - 196$

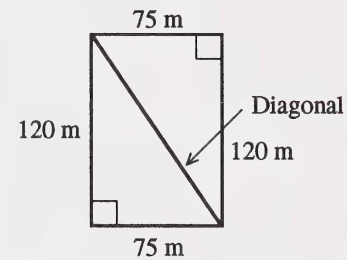
$x^2 = 6693$

$\sqrt{x^2} = \sqrt{6693}$

$x \doteq 81.810\,757\,24$

$\doteq 81.8$ units

2. Find the diagonal of the rectangular field first.



$x^2 = 120^2 + 75^2$

$x^2 = 14\,400 + 5625$

$x^2 = 20\,025$

$\sqrt{x^2} = \sqrt{20\,025}$

$x \doteq 141.509\,717$

$\doteq 142$ m

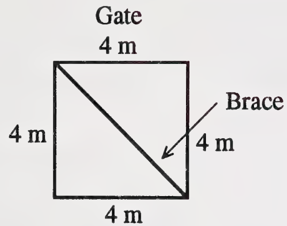
The diagonal is about 142 m.

The distance along the length and width of the field is $120 \text{ m} + 75 \text{ m} = 195 \text{ m}$.

The difference is $195 \text{ m} - 142 \text{ m} \doteq 53 \text{ m}$.

It is about 53 m closer if you go from one corner of the field to the opposite corner.

3. Let x be the length of the brace.



$$x^2 = 4^2 + 4^2$$
$$x^2 = 16 + 16$$
$$x^2 = 32$$
$$\sqrt{x^2} = \sqrt{32}$$
$$x \doteq 5.656\,854\,249$$
$$x \doteq 5.7 \text{ m}$$

The diagonal brace is about 5.7 m.

Extensions

1. a.
$$x^2 = 3^2 + 3^2$$
$$x^2 = 9 + 9$$
$$x^2 = 18$$
$$\sqrt{x^2} = \sqrt{18}$$
$$x = \sqrt{9 \times 2}$$
$$= 3\sqrt{2}$$

b.
$$x^2 = 2^2 + 2^2$$
$$x^2 = 4 + 4$$
$$x^2 = 8$$
$$\sqrt{x^2} = \sqrt{8}$$
$$x = \sqrt{4 \times 2}$$
$$= 2\sqrt{2}$$

2. a.
$$\left(8\sqrt{2}\right)^2 = x^2 + 8^2$$
$$64 \times 2 = x^2 + 64$$
$$128 = x^2 + 64$$
$$x^2 = 128 - 64$$
$$x^2 = 64$$
$$\sqrt{x^2} = \sqrt{64}$$
$$x = 8$$

b.
$$\left(9\sqrt{2}\right)^2 = 9^2 + x^2$$
$$81 \times 2 = 81 + x^2$$
$$162 = 81 + x^2$$
$$x^2 = 162 - 81$$
$$x^2 = 81$$
$$\sqrt{x^2} = \sqrt{81}$$
$$x = 9$$

3. a.

LS	RS
6^2	3^2+3^2
36	$9+9$
36	18

LS $\neq$ RS

It is not a right triangle.

b.

LS	RS
$\left(\sqrt{113}\right)^2$	7^2+8^2
113	$49+64$
113	113

LS $=$ RS

It is a right triangle.

4. a.

LS	RS
$\left(\sqrt{5}\right)^2$	$\left(\sqrt{3}\right)^2+\left(\sqrt{2}\right)^2$
5	$3+2$
5	5

LS $=$ RS

It is a right triangle.

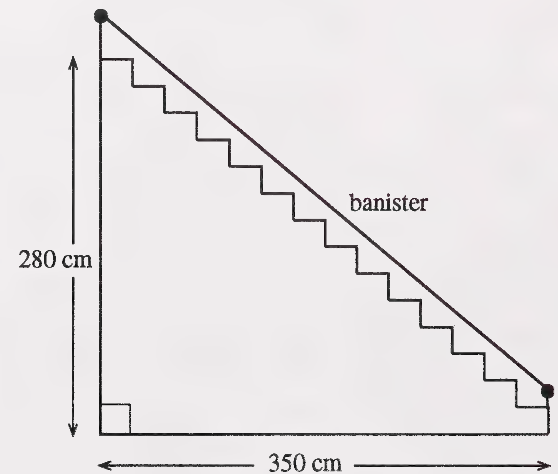
b.

LS	RS
$\left(2\sqrt{7}\right)^2$	4^2+3^2
4×7	$16+9$
28	25

LS $\neq$ RS

It is not a right triangle.

5.



Note: The length of the banister is the same as diagonal distance from the bottom of the first step to the end of the top step.

The length of staircase is $14\times25=350$ cm.

The height of staircase is $14\times20=280$ cm.

Let x be the length of the banister.

$$\begin{aligned}
x^2 &= 350^2 + 280^2 \\
x^2 &= 122\,500 + 78\,400 \\
x^2 &= 200\,900 \\
\sqrt{x^2} &= \sqrt{200\,900} \\
x &= \sqrt{2 \times 2 \times 5 \times 5 \times 7 \times 7 \times 41} \\
&= \sqrt{2^2 \times 5^2 \times 7^2 \times 41} \\
&= 2 \times 5 \times 7\sqrt{41} \\
&= 70\sqrt{41}
\end{aligned}$$

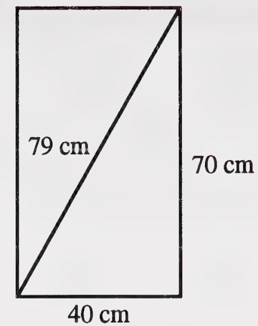
Find the prime factorization for 200 900.

$$\begin{aligned}
200\,900 &= 2 \times 100\,450 \\
&= 2 \times 2 \times 50\,225 \\
&= 2 \times 2 \times 5 \times 10\,045 \\
&= 2 \times 2 \times 5 \times 5 \times 2009 \\
&= 2 \times 2 \times 5 \times 5 \times 7 \times 287 \\
&= 2 \times 2 \times 5 \times 5 \times 7 \times 7 \times 41
\end{aligned}$$

The length of the banister as a radical is $70\sqrt{41}$ cm.

The length of the banister as a decimal number is about 448.2 cm rounded to one decimal place.

6.



The diagonal of the rectangle is the hypotenuse of the two right triangles formed when the rectangle is divided into equal sections.

LS		RS
c^2		$a^2 + b^2$
79^2		$70^2 + 40^2$
6241		4900 + 1600
6241		6500
LS	$\neq$	RS

Therefore, the corners of the square are not right angles.

Exploring Topic 2

Activity 1

Solve problems involving the sum of the angles of a triangle.

1. $$\begin{aligned} x+93^\circ+63^\circ &= 180^\circ \\ x+156^\circ &= 180^\circ \\ x &= 180^\circ-156^\circ \\ &= 24^\circ \end{aligned}$$

2. $$\begin{aligned} x+119^\circ+32^\circ &= 180^\circ \\ x+151^\circ &= 180^\circ \\ x &= 180^\circ-151^\circ \\ &= 29^\circ \end{aligned}$$

3. The supplement of 115° is $180^\circ-115^\circ=65^\circ$.

$$\begin{aligned} x+65^\circ+87^\circ &= 180^\circ \\ x+152^\circ &= 180^\circ \\ x &= 180^\circ-152^\circ \\ &= 28^\circ \end{aligned}$$

4. The supplement of 70° is $180^\circ-70^\circ=110^\circ$.

$$\begin{aligned} x+35^\circ+110^\circ &= 180^\circ \\ x+145^\circ &= 180^\circ \\ x &= 180^\circ-145^\circ \\ &= 35^\circ \end{aligned}$$

5. The supplement of 94° is $80^\circ-94^\circ=86^\circ$.

$$\begin{aligned} x+80^\circ+86^\circ &= 180^\circ \\ x+166^\circ &= 180^\circ \\ x &= 180^\circ-166^\circ \\ &= 14^\circ \end{aligned}$$

6. The supplement of 170° is $180^\circ-170^\circ=10^\circ$.

$$\begin{aligned} x+37^\circ+10^\circ &= 180^\circ \\ x+47^\circ &= 180^\circ \\ x &= 180^\circ-47^\circ \\ &= 133^\circ \end{aligned}$$

Activity 2

Recognize and write the relationship between similar triangles.

1. a. $\overline{DE}$ is proportional to $\underline{\overline{PQ}}$.
   $\overline{EF}$ is proportional to $\underline{\overline{QR}}$.
   $\overline{DF}$ is proportional to $\underline{\overline{PR}}$.

   $\angle A = \underline{\angle L}$

   $\angle B = \underline{\angle M}$

   $\angle C = \underline{\angle N}$

2. Check to see if the specified sides are proportional.

LS	RS
$\frac{AD}{AC}$	$\frac{AE}{AB}$
$\frac{2.4}{3.45}$	$\frac{3.2}{4.6}$
2.4×4.6	3.2×3.45
11.04	11.04

LS = RS

Therefore, the cross products are equal. This means that the sides are proportional and that the triangles are similar.

3. a. Since $\Delta RST \sim \Delta UVW$, then

$$\begin{array}{lll} \overline{RS} \leftrightarrow \overline{UV} & \text{and} & \angle R = \angle U \\ \overline{ST} \leftrightarrow \overline{VW} & & \angle S = \angle V \\ \overline{RT} \leftrightarrow \overline{UW} & & \angle T = \angle W \end{array}$$

   b. Since $\Delta DEF \sim \Delta DCB$, then

$$\begin{array}{lll} \overline{DE} \leftrightarrow \overline{DC} & \text{and} & \angle EDF = \angle CDB \\ \overline{EF} \leftrightarrow \overline{CB} & & \angle E = \angle C \\ \overline{DF} \leftrightarrow \overline{DB} & & \angle F = \angle B \end{array}$$

4. $\frac{XZ}{RT} = \frac{XY}{RS}$

$\frac{x}{14} = \frac{18}{10}$

$10x = 252$

$\frac{10x}{10} = \frac{252}{10}$

$x = 25\frac{1}{5}$ or 25.2

$\frac{ST}{YZ} = \frac{RS}{XY}$

$\frac{y}{24} = \frac{10}{18}$

$18y = 240$

$\frac{18y}{18} = \frac{240}{18}$

$y = 13\frac{1}{3}$ or $13.\overline{3}$

5. Solve for x.

$$\frac{x}{18}=\frac{15}{10}$$
$$10x=270$$
$$\frac{10x}{10}=\frac{270}{10}$$
$$x=27$$

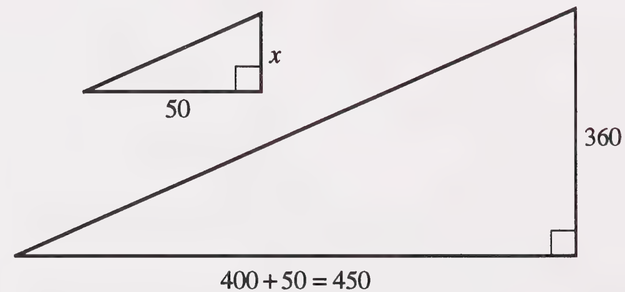
The value of x is 27.

Solve for y.

$$\frac{y}{8}=\frac{10}{15}$$
$$15y=80$$
$$\frac{15y}{15}=\frac{80}{15}$$
$$y \doteq 5.\overline{3} \text{ or } 5\frac{1}{3}$$

The value of y is $5.\overline{3}$ or $5\frac{1}{3}$.

6. Look at the two triangles separately.



$$\frac{x}{360}=\frac{50}{450}$$
$$450x=18\,000$$
$$\frac{450x}{450}=\frac{18\,000}{450}$$
$$x=40$$

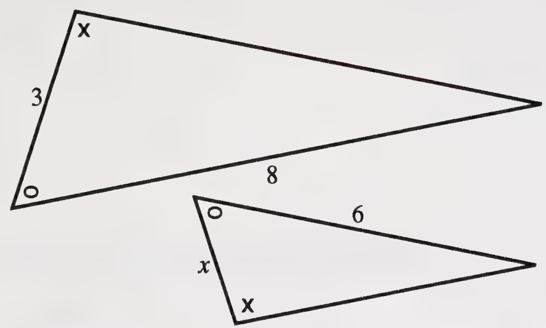
The value of x is 40.

Verification:

LS	RS
$\frac{x}{360}$	$\frac{50}{450}$
$\frac{40}{360}$	$\frac{50}{450}$
40×450	360×50
18 000	18 000

LS = RS

7. Look at the two triangles separately.



$$\frac{x}{3}=\frac{6}{8}$$
$$8x=18$$
$$x=\frac{18}{8}$$
$$=\frac{9}{4}$$
$$=2\frac{1}{4}$$

Verification:

LS	RS
$\frac{x}{3}$	$\frac{6}{8}$
$\frac{2\frac{1}{4}}{3}$	$\frac{6}{8}$
$2\frac{1}{4}\times 8$	3×8
$\frac{9}{4}\times 8$	18
9×2	18
18	18

LS = RS

Activity 3

Solve problems by using properties of similar triangles.

1. Let the height of the stack be h.

$$\frac{15}{90}=\frac{10}{h}$$
$$15h=90\times 10$$
$$15h=900$$
$$h=\frac{900}{15}$$
$$=60$$

The incinerator is 60 m high.

2. Let the height of the tree be h.

$$\frac{1.6}{2.5}=\frac{h}{30}$$
$$2.5h=30\times 1.6$$
$$2.5h=48$$
$$h=\frac{48}{2.5}$$
$$=19.2$$

The height of the tree is 19.2 m.

3. $\Delta CED \sim \Delta CBA$

$$\begin{aligned} \frac{ED}{BA} &= \frac{CD}{CA} \\ \frac{c}{4.4} &= \frac{32.5}{7.2} \\ 7.2c &= 143 \\ \frac{7.2c}{7.2} &= \frac{143}{7.2} \\ c &\doteq 19.86111111 \\ &\doteq 20 \end{aligned}$$

The width of the river is approximately 20 m.

4. $$\begin{aligned} \frac{d}{95.7} &= \frac{70.2}{50.4} \\ 50.4d &= 6718.14 \\ \frac{50.4d}{50.4} &= \frac{6718.14}{50.4} \\ d &\doteq 133.296\,4286 \\ &\doteq 133 \end{aligned}$$

The distance across the pond is about 133 m.

Extra Help

1. a. $$\begin{aligned} x + 52° + 32° &= 180° \\ x + 84° &= 180° \\ x &= 180° - 84° \\ &= 96° \end{aligned}$$

The value of x is 96°.

b. $$\begin{aligned} x + 35° + 120° &= 180° \\ x + 155° &= 180° \\ x &= 180° - 155° \\ &= 25° \end{aligned}$$

The value of x is 25°.

c. $$\begin{aligned} x &= 180° - 126° \\ &= 54° \end{aligned}$$

The value of x is 54°.

d. $$\begin{aligned} x + 56° + 40° &= 180° \\ x + 96° &= 180° \\ x &= 180° - 96° \\ &= 84° \end{aligned}$$

The value of x is 84°.

e. $w = 180° - 140°$
$= 40°$

$$\begin{aligned} x + 82° + 40° &= 180° \\ x + 122° &= 180° \\ x &= 180° - 122° \\ &= 58° \end{aligned}$$

Since y and $\angle ACB$ are opposite angles, $y = 82°$.

$z = 180° - 140°$
$= 40°$

$$\begin{aligned} v + 82° + 40° &= 180° \\ v + 122° &= 180° \\ v &= 180° - 122° \\ &= 58° \end{aligned}$$

The values of the missing measures are $w = 40°$, $x = 58°$, $y = 82°$, $z = 40°$, and $v = 58°$.

f. $w = 180° - 70°$
$= 110°$

$$\begin{aligned} y + 30° + 70° &= 180° \\ y + 100° &= 180° \\ y &= 180° - 100° \\ &= 80° \end{aligned}$$

Since x and y are opposite angles, $x = y = 80°$..

$$\begin{aligned} z + 70° + 80° &= 180° \\ z + 150° &= 180° \\ z &= 180° - 150° \\ &= 30° \end{aligned}$$

The values of the missing measures are $w = 110°$, $y = 80°$, $x = 80°$, and $z = 30°$.

2. a. $\Delta ABC \sim \Delta DEF$

$$\frac{9}{12} = \frac{x}{20} \qquad \frac{3}{y} = \frac{9}{12}$$

$$12x = 9 \times 20 \qquad 9y = 3 \times 12$$

$$12x = 180 \qquad 9y = 36$$

$$\frac{12x}{12} = \frac{180}{12} \qquad \frac{9y}{9} = \frac{36}{9}$$

$$x = 15 \qquad y = 4$$

The value of x is 15, and the value of y is 4.

b. $\Delta ABC \sim \Delta DEF$

$\frac{x}{18}=\frac{5}{6}$

$6x=5\times18$

$6x=90$

$\frac{6x}{6}=\frac{90}{6}$

$x=15$

$\frac{10}{y}=\frac{5}{6}$

$5y=6\times10$

$5y=60$

$\frac{5y}{5}=\frac{60}{5}$

$y=12$

The value of x is 15, and the value of y is 12.

c. $\Delta ABC \sim \Delta DEF$

$\frac{21}{7}=\frac{24}{d}$

$21d=24\times7$

$21d=168$

$\frac{21d}{21}=\frac{168}{21}$

$d=8$

$\frac{21}{7}=\frac{b}{6}$

$7b=21\times6$

$7b=126$

$\frac{7b}{7}=\frac{126}{7}$

$b=18$

The value of d is 8, and the value of b is 18.

d. $\Delta ABC \sim \Delta DEF$

$\frac{x}{15}=\frac{12}{9}$

$9x=15\times12$

$9x=180$

$\frac{9x}{9}=\frac{180}{9}$

$x=20$

$\frac{y}{12}=\frac{12}{9}$

$9y=12\times12$

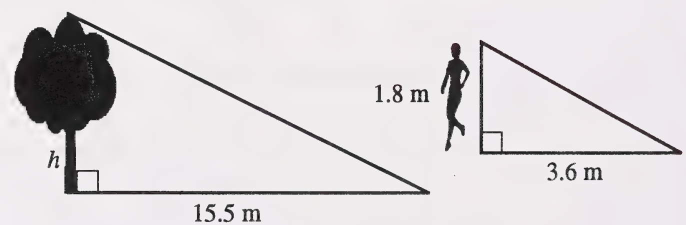
$9y=144$

$\frac{9y}{9}=\frac{144}{9}$

$y=16$

The value of x is 20, and the value of y is 16.

3. Let h be the height of the tree.



$\frac{h}{1.8}=\frac{15.5}{3.6}$

$3.6h=15.5\times1.8$

$3.6h=27.9$

$\frac{3.6h}{3.6}=\frac{27.9}{3.6}$

$h=7.75\text{ m}$

The height of the tree is 7.75 m.

Extensions

1. a.

$$\begin{aligned} \angle A+\angle B+\angle C &= 180^\circ \\ 4x+2x+3+4x-3 &= 180 \\ 10x &= 180 \\ \frac{10x}{10} &= \frac{180}{10} \\ x &= 18 \end{aligned}$$

$$\begin{aligned} \angle A &= 4x \\ &= 4(18) \\ &= 72^\circ \end{aligned} \qquad \begin{aligned} \angle B &= 2x+3 \\ &= 2(18)+3 \\ &= 36+3 \\ &= 39^\circ \end{aligned} \qquad \begin{aligned} \angle C &= 4x-3 \\ &= 4(18)-3 \\ &= 72-3 \\ &= 69^\circ \end{aligned}$$

$$\begin{aligned} \text{Total} &= 72^\circ+39^\circ+69^\circ \\ &= 180^\circ \end{aligned}$$

b.

$$\begin{aligned} \angle A+\angle B+\angle C &= 180^\circ \\ x+3x+17+2x+7 &= 180^\circ \\ 6x+24 &= 180 \\ 6x &= 180-24 \\ 6x &= 156 \\ \frac{6x}{6} &= \frac{156}{6} \\ x &= 26 \end{aligned}$$

$$\angle A = 26^\circ \qquad \begin{aligned} \angle B &= 3x+17 \\ &= 3(26)+17 \\ &= 78+17 \\ &= 95^\circ \end{aligned} \qquad \begin{aligned} \angle C &= 2x+7 \\ &= 2(26)+7 \\ &= 52+7 \\ &= 59^\circ \end{aligned}$$

$$\begin{aligned} \text{Total} &= 26^\circ+95^\circ+59^\circ \\ &= 180^\circ \end{aligned}$$

2. a.

$$\begin{aligned} \frac{AB}{DE} &= \frac{AC}{DF} \\ \frac{16}{11} &= \frac{18}{x+5} \\ 16(x+5) &= 11\times 18 \\ 16x+80 &= 198 \\ 16x &= 198-80 \\ 16x &= 118 \\ \frac{16x}{16} &= \frac{118}{16} \\ x &= 7.375 \end{aligned}$$

The length of the side represented by $(x+5)$ is $7.375+5=12.375 \doteq 12.4$ units.

$$\frac{BC}{EF}=\frac{AB}{DE}$$

$$\frac{y-3}{16}=\frac{16}{11}$$

$$11(y-3)=16\times16$$

$$11y-33=256$$

$$11y=289$$

$$\frac{11y}{11}=\frac{289}{11}$$

$$y=26.\overline{27}$$

The length of the side represented by $y-3$ is $26.\overline{27}-3=23.\overline{27}\doteq23.3$ units.

b.

$$\frac{AB}{DF}=\frac{BC}{FE}$$

$$\frac{x-3}{10}=\frac{5}{8}$$

$$8(x-3)=5\times10$$

$$8x-24=50$$

$$8x=50+24$$

$$8x=74$$

$$\frac{8x}{8}=\frac{74}{8}$$

$$x=9.25$$

The length of the side represented by $x-3$ is $9.25-3=6.25\doteq6.3$ units.

$$\frac{DE}{AC}=\frac{FE}{BC}$$

$$\frac{y+4}{12}=\frac{8}{5}$$

$$5(y+4)=8\times12$$

$$5y+20=96$$

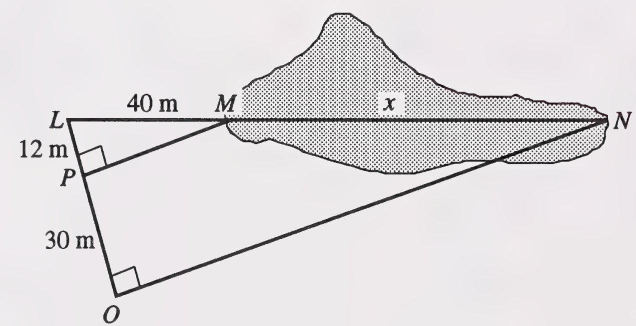
$$5y=76$$

$$\frac{5y}{5}=\frac{76}{5}$$

$$y=15.2$$

The length of the $(y+4)$ side is $15.2+4=19.2$ units.

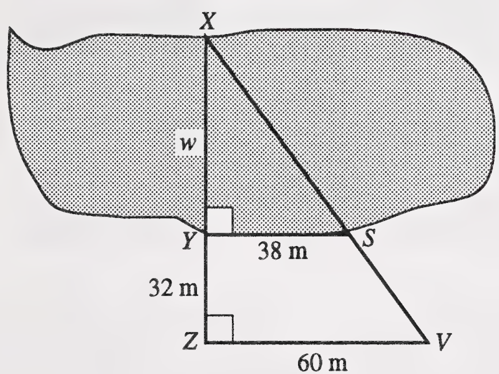
3. a.



$$\frac{x+40}{40}=\frac{42}{12}$$
$$12(x+40)=42\times 40$$
$$12x+480=1680$$
$$12x=1200$$
$$\frac{12x}{12}=\frac{1200}{12}$$
$$x=100$$

The length of the lake is 100 m.

b.



$$\frac{w}{32+w}=\frac{38}{60}$$
$$60w=38(32+w)$$
$$60w=1216+38w$$
$$60w-38w=1216$$
$$22w=1216$$
$$\frac{22w}{22}=\frac{1216}{22}$$
$$w=55.\overline{27}$$
$$\doteq 55$$

The width of the river is approximately 55 m.

Exploring Topic 3

Activity 1

Define the terms opposite side, adjacent side, and hypotenuse.

1. a. The side opposite θ is $\overline{AC}$ or b.
   The side adjacent to θ is $\overline{BC}$ or a.

   b. The side opposite θ is $\overline{BC}$ or a.
   The side adjacent to θ is $\overline{AC}$ or b.

2. a. The side opposite θ is $\overline{YZ}$ or x.
The side adjacent to θ is $\overline{XZ}$ or y.

b. The side opposite θ is $\overline{XZ}$ or y.
The side adjacent to θ is $\overline{YZ}$ or x.

Activity 2

Use the properties of similar triangles to develop tangent, sine, and cosine ratios.

1. a. $\tan\ \theta = \frac{b}{a}$ or $\frac{AC}{BC}$

$\sin\ \theta = \frac{b}{c}$ or $\frac{AC}{AB}$

$\cos\ \theta = \frac{a}{c}$ or $\frac{BC}{AB}$

b. $\tan\ \theta = \frac{a}{b}$ or $\frac{BC}{AC}$

$\sin\ \theta = \frac{a}{c}$ or $\frac{BC}{AB}$

$\cos\ \theta = \frac{b}{c}$ or $\frac{AC}{AB}$

2. a. $\tan\ \theta = \frac{d}{e}$ or $\frac{EF}{DF}$

$\sin\ \theta = \frac{d}{f}$ or $\frac{EF}{DE}$

$\cos\ \theta = \frac{e}{f}$ or $\frac{DF}{DE}$

b. $\tan\ \theta = \frac{e}{d}$ or $\frac{DF}{EF}$

$\sin\ \theta = \frac{e}{f}$ or $\frac{DF}{DE}$

$\cos\ \theta = \frac{d}{f}$ or $\frac{EF}{DE}$

3. $\tan\ \theta = \frac{8}{6} = \frac{4}{3} \doteq 1.33$

$\sin\ \theta = \frac{8}{10} = \frac{4}{5} = 0.8$

$\cos\ \theta = \frac{6}{10} = \frac{3}{5} = 0.6$

$\tan \propto = \frac{6}{8} = \frac{3}{4} = 0.75$

$\sin \propto = \frac{6}{10} = \frac{3}{5} = 0.6$

$\cos \propto = \frac{8}{10} = \frac{4}{5} = 0.8$

4. $\tan\ \theta = \frac{24}{10} = \frac{12}{5} = 2.4$

$\sin\ \theta = \frac{24}{26} = \frac{12}{13} \doteq 0.92$

$\cos\ \theta = \frac{10}{26} = \frac{5}{13} \doteq 0.38$

$\tan \propto = \frac{10}{24} = \frac{5}{12} \doteq 0.42$

$\sin \propto = \frac{10}{26} = \frac{5}{13} \doteq 0.38$

$\cos \propto = \frac{24}{26} = \frac{12}{13} \doteq 0.92$

Activity 3

Use a calculator to determine the trigonometric ratio given the measure of an angle, and to determine the measure of an angle given a particular ratio.

1. a.

B	11°	21°	31°	41°	51°	61°
$\tan B$	0.1944	0.3839	0.6009	0.8693	1.2349	1.8040

b. The tangent values **increase** as the measure of the angle increases.

2. a.

B	5°	10°	15°	20°	25°	30°
$\sin B$	0.0872	0.1736	0.2588	0.3420	0.4226	0.5000

b. The sine values **increase** as the measure of the angle increases.

3. a.

B	14°	24°	27°	37°	41°	51°
$\cos B$	0.9703	0.9135	0.8910	0.7986	0.7547	0.6293

b. The cosine values **decrease** as the measure of the angle increases.

4. $\tan A = \frac{11}{15}$

Enter	Display
11	11
÷	11
15	15
=	0.733333333
INV	0.733333333
tan	36.25383774

Therefore, $\angle A \doteq 36°$.

5. $\cos B = \frac{16}{23}$

Enter	Display
16	16
÷	16
23	23
=	0.695652173
INV	0.695652173
cos	45.92079015

Therefore, $\angle B \doteq 46°$.

6. $\sin A = \frac{24}{54}$

Enter	Display
24	24
$\boxed{\div}$	24
54	54
$\boxed{=}$	0.444444444
$\boxed{\text{INV}}$	0.444444444
$\boxed{\text{sin}}$	26.38779996

Therefore, $\angle A \doteq 26°$.

7. $\sin C = \frac{2}{7}$

Enter	Display
2	2
$\boxed{\div}$	2
7	7
$\boxed{=}$	0.285714285
$\boxed{\text{INV}}$	0.285714285
$\boxed{\text{sin}}$	16.6015496

Therefore, $\angle C \doteq 17°$.

8. $\tan B = \frac{7}{3}$

Enter	Display
7	7
$\boxed{\div}$	7
3	3
$\boxed{=}$	2.333333333
$\boxed{\text{INV}}$	2.333333333
$\boxed{\text{tan}}$	66.80140949

Therefore, $\angle B = 67°$.

Activity 4

Determine the sine, cosine, and tangent ratios within right triangles, given the measures of any two sides.

1. a. $\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$

$= \frac{8}{12}$

$= \frac{2}{3}$

b. $\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$

$= \frac{24}{32}$

$= \frac{3}{4}$

2. a.. $\sin D = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{20}{25} = \frac{4}{5}$

$\sin F = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{15}{25} = \frac{3}{5}$

$\cos D = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{15}{25} = \frac{3}{5}$

$\cos F = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{20}{25} = \frac{4}{5}$

b. $\sin D = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{12}{15} = \frac{4}{5}$

$\sin F = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{9}{15} = \frac{3}{5}$

$\cos D = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{9}{15} = \frac{3}{5}$

$\cos F = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{12}{15} = \frac{4}{5}$

3. a. $\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$

$= \frac{10}{14}$

$= \frac{5}{7}$

$\doteq 0.714\,285\,714$

$\doteq 0.7143$

b. $\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$

$= \frac{7}{16}$

$= 0.4375$

4. a. Find c first.

$$c^2 = a^2 + b^2$$
$$c^2 = 24^2 + 18^2$$
$$c^2 = 576 + 324$$
$$c^2 = 900$$
$$\sqrt{c^2} = \sqrt{900}$$
$$c = 30$$

$\sin A = \frac{24}{30} = \frac{4}{5} = 0.8000$

$\cos A = \frac{18}{30} = \frac{3}{5} = 0.6000$

$\tan A = \frac{24}{18} = \frac{4}{3} \doteq 1.3333$

$$\sin B = \frac{18}{30} = \frac{3}{5} = 0.6000 \qquad \cos B = \frac{24}{30} = \frac{4}{5} = 0.8000 \qquad \tan B = \frac{18}{24} = \frac{3}{4} = 0.7500$$

b. Find a first.

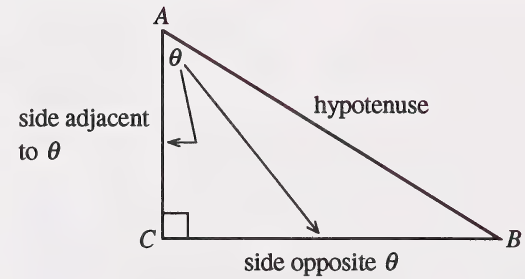
$$c^2 = a^2 + b^2$$
$$40^2 = a^2 + 32^2$$
$$1600 = a^2 + 1024$$
$$a^2 = 576$$
$$\sqrt{a^2} = \sqrt{576}$$
$$a = 24$$

$$\sin A = \frac{24}{40} = \frac{3}{5} = 0.6000 \qquad \cos A = \frac{32}{40} = \frac{4}{5} = 0.8000 \qquad \tan A = \frac{24}{32} = \frac{3}{4} = 0.7500$$

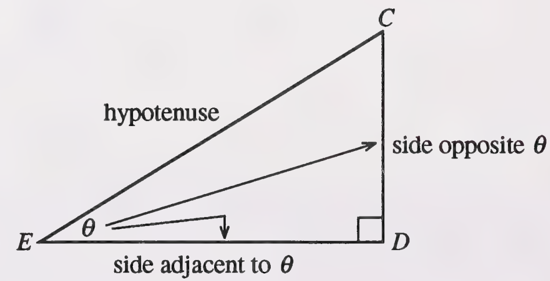
$$\sin B = \frac{32}{40} = \frac{4}{5} = 0.8000 \qquad \cos B = \frac{24}{40} = \frac{3}{5} = 0.6000 \qquad \tan B = \frac{32}{24} = \frac{4}{3} \doteq 1.3333$$

Extra Help

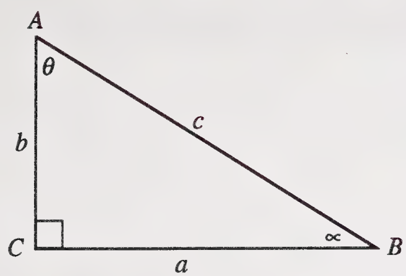
1.



2.



3.

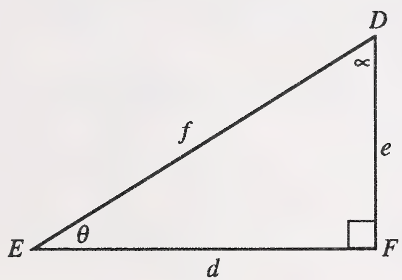


$\tan\,\theta = \frac{a}{b}$ $\quad$ $\tan\,\propto = \frac{b}{a}$

$\sin\,\theta = \frac{a}{c}$ $\quad$ $\sin\,\propto = \frac{b}{c}$

$\cos\,\theta = \frac{b}{c}$ $\quad$ $\cos\,\propto = \frac{a}{c}$

4.



$\tan\,\theta = \frac{e}{d}$ $\quad$ $\tan\,\propto = \frac{d}{e}$

$\sin\,\theta = \frac{e}{f}$ $\quad$ $\sin\,\propto = \frac{d}{f}$

$\cos\,\theta = \frac{d}{f}$ $\quad$ $\cos\,\propto = \frac{e}{f}$

Extensions

1. $\tan\,\theta = \frac{4.2}{4.7} \doteq 0.8936$ $\quad$ $\sin\,\theta = \frac{4.2}{6.3} \doteq 0.6667$ $\quad$ $\cos\,\theta = \frac{4.7}{6.3} \doteq 0.7460$

   $\tan\,\propto = \frac{4.7}{4.2} \doteq 1.1190$ $\quad$ $\sin\,\propto = \frac{4.7}{6.3} \doteq 0.7460$ $\quad$ $\cos\,\propto = \frac{4.2}{6.3} \doteq 0.6667$

2. $\tan\,\theta = \frac{4\frac{2}{3}}{3\frac{3}{5}} = \frac{\frac{14}{3}}{\frac{18}{5}} \doteq 1.2963$ $\quad$ $\sin\,\theta = \frac{4\frac{2}{3}}{5.9} = \frac{\frac{14}{3}}{5.9} \doteq 0.7910$ $\quad$ $\cos\,\theta = \frac{3\frac{3}{5}}{5.9} = \frac{\frac{18}{5}}{5.9} \doteq 0.6102$

   $\tan\,\propto = \frac{3\frac{3}{5}}{4\frac{2}{3}} = \frac{\frac{18}{5}}{\frac{14}{3}} \doteq 0.7714$ $\quad$ $\sin\,\propto = \frac{3\frac{3}{5}}{5.9} = \frac{\frac{18}{5}}{5.9} \doteq 0.6102$ $\quad$ $\cos\,\propto = \frac{4\frac{2}{3}}{5.9} = \frac{\frac{14}{3}}{5.9} \doteq 0.7910$

3. $\tan\ \theta = \frac{4\sqrt{6}}{2\sqrt{5}}$

$= \frac{2\sqrt{6}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$

$= \frac{2\sqrt{30}}{5}$

$\tan\ \propto = \frac{2\sqrt{5}}{4\sqrt{6}}$

$= \frac{\sqrt{5}}{2\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}}$

$= \frac{\sqrt{30}}{12}$

$\sin\ \theta = \frac{4\sqrt{6}}{2\sqrt{29}}$

$= \frac{2\sqrt{6}}{\sqrt{29}} \times \frac{\sqrt{29}}{\sqrt{29}}$

$= \frac{2\sqrt{174}}{29}$

$\sin\ \propto = \frac{2\sqrt{5}}{2\sqrt{29}}$

$= \frac{\sqrt{5}}{\sqrt{29}} \times \frac{\sqrt{29}}{\sqrt{29}}$

$= \frac{\sqrt{145}}{29}$

$\cos\ \theta = \frac{2\sqrt{5}}{2\sqrt{29}}$

$= \frac{\sqrt{5}}{\sqrt{29}} \times \frac{\sqrt{29}}{\sqrt{29}}$

$= \frac{\sqrt{145}}{29}$

$\cos\ \propto = \frac{4\sqrt{6}}{2\sqrt{29}}$

$= \frac{2\sqrt{6}}{\sqrt{29}} \times \frac{\sqrt{29}}{\sqrt{29}}$

$= \frac{2\sqrt{174}}{29}$

Exploring Topic 4

Activity 1

Apply trigonometric ratios to determine the measure of an angle of a given right triangle.

1. a. The only trig ratio that can be used to find $\angle A$ is the sine ratio.

$\sin\ A = \frac{17}{42}$

Enter	Display
17	17
$\boxed{\div}$	17
42	42
$\boxed{=}$	0.404761904
$\boxed{\text{INV}}$	0.404761904
$\boxed{\sin}$	23.87620773

To the nearest degree, $\angle A = 24°$.

b. The only trig ratio that can be used to find $\angle A$ is the tangent ratio.

$$\tan A = \frac{43}{32}$$

Enter	Display
43	43
÷	43
32	32
=	1.34375
INV	1.34375
tan	53.34389159

To the nearest degree, $\angle A = 53°$.

2. a. The only trig ratio that can be used to find $\angle B$ is the cosine ratio.

$$\cos B = \frac{27}{42}$$

Enter	Display
27	27
÷	27
42	42
=	0.642857142
INV	0.642857142
cos	49.99479912

To the nearest degree, $\angle B = 50°$.

b. The only trig ratio that can be used to find $\angle B$ is the tangent ratio.

$$\tan B = \frac{27}{17}$$

Enter	Display
27	27
÷	27
17	17
=	1.588235294
INV	1.588235294
tan	57.80426607

To the nearest degree, $\angle B = 58°$.

Activity 2

Apply trigonometric ratios to determine the length of a side of a given right triangle.

1. Since you want a side that is opposite $\angle A$, and since 34 is adjacent to $\angle A$, use the tangent ratio.

$$\tan A = \frac{a}{34}$$

$$\tan 43° = \frac{a}{34}$$

$$\begin{aligned} a &= 34(\tan 43°) \\ &\doteq 34 \times 0.932\,515\,086 \\ &\doteq 31.705\,512\,93 \\ &\doteq 32 \end{aligned}$$

The measure of a is approximately 32 units.

2. Since b is opposite $\angle B$ and 12 is adjacent to $\angle B$, use the tangent ratio.

$$\tan B = \frac{b}{12}$$

$$\tan 39° = \frac{b}{12}$$

$$\begin{aligned} b &= 12(\tan 39°) \\ &\doteq 12 \times 0.809\,784\,033 \\ &\doteq 9.717\,408\,398 \\ &\doteq 10 \end{aligned}$$

The measure for b is approximately 10 units.

3. Choose the tangent or sine ratio first; then check using the other one.

Use tan find a.

$$\begin{aligned}\tan A &= \frac{a}{42}\\ \tan 51° &= \frac{a}{42}\\ a &= 42(\tan 51°)\\ &\doteq 42 \times 1.234\,897\,157\\ &\doteq 51.865\,680\,57\\ &\doteq 52 \text{ units}\end{aligned}$$

Check by using sine.

$$\begin{aligned}\sin A &= \frac{a}{67}\\ \sin 51° &= \frac{a}{67}\\ a &= 67(\sin 51°)\\ &\doteq 67 \times 0.777\,145\,961\\ &\doteq 52.068\,779\,42\\ &\doteq 52 \text{ units}\end{aligned}$$

Both are the same no matter which trig ratio is used.

This solution shows that in many instances there are more than one way to solve a problem.

You can also use the Pythagorean theorem to solve this problem.

$$\begin{aligned}a^2 + b^2 &= c^2\\ a^2 + 42^2 &= 67^2\\ a^2 &= 4489 - 1764\\ a^2 &= 2725\\ a &\doteq 52.201\,532\,54\\ &\doteq 52 \text{ units}\end{aligned}$$

4. Choose the tangent or sine ratio first; then check using the other one.

Use tan find b.

$$\begin{aligned}\tan B &= \frac{b}{31}\\ \tan 63° &= \frac{b}{31}\\ b &= 31(\tan 63°)\\ &\doteq 31 \times 1.962\,610\,505\\ &\doteq 60.840\,925\,67\\ &\doteq 61 \text{ units}\end{aligned}$$

Check by using sine.

$$\begin{aligned}\sin B &= \frac{b}{68}\\ \sin 63° &= \frac{b}{68}\\ b &= 68(\sin 63°)\\ &\doteq 68 \times 0.891\,006\,524\\ &\doteq 60.588\,443\,65\\ &\doteq 61 \text{ units}\end{aligned}$$

The result is the same no matter which trig ratio is used.

You can also use the Pythagorean theorem.

$$\begin{aligned}a^2 + b^2 &= c^2\\ 31^2 + b^2 &= 68^2\\ b^2 &= 4624 - 961\\ b^2 &= 3663\\ b &\doteq 60.522\,723\\ &\doteq 61 \text{ units}\end{aligned}$$

Activity 3

Apply trigonometric ratios to solve right triangles.

1. a.

$$\begin{aligned}\tan A &= \frac{17}{13}\\ \tan A &\doteq 1.307\,692\,308\\ \angle A &\doteq 52.594\,643\,37\\ &\doteq 53°\end{aligned}$$

$$\begin{aligned}\tan B &= \frac{13}{17}\\ \tan B &\doteq 0.764\,705\,882\\ \angle B &\doteq 37.405\,356\,63\\ &\doteq 37°\end{aligned}$$

The measure of $\angle B$ can be found the following way as well.

$$\begin{aligned}\angle A + \angle B + \angle C &= 180°\\ 53° + \angle B + 90° &\doteq 180°\\ 143° + \angle B &\doteq 180°\\ \angle B &\doteq 180° - 143°\\ &\doteq 37°\end{aligned}$$

Use the theorem of Pythagoras to find c.

$$\begin{aligned} c^2 &= a^2 + b^2 \\ c^2 &= 17^2 + 13^2 \\ c^2 &= 289 + 169 \\ c^2 &= 458 \\ \sqrt{c^2} &= \sqrt{458} \\ c &\doteq 21.400\,934\,56 \\ &\doteq 21 \text{ units} \end{aligned}$$

The summary of triangle ABC is as follows:

$\angle A \doteq 53°$	$a = 17$ units
$\angle B \doteq 37°$	$b = 13$ units
$\angle C = 90°$	$c \doteq 21$ units

b. $\sin A = \dfrac{a}{22}$

$$\begin{aligned} \sin 46° &= \frac{a}{22} \\ a &= 22(\sin 46°) \\ &= 22 \times 0.719\,3398 \\ &\doteq 15.825\,475\,61 \\ &\doteq 16 \text{ units} \end{aligned}$$

$$\begin{aligned} \cos A &= \frac{b}{22} \\ \cos 46° &= \frac{b}{22} \\ b &= 22(\cos 46°) \\ &\doteq 22 \times 0.694\,658\,37 \\ &\doteq 15.282\,484\,15 \\ &\doteq 15 \text{ units} \end{aligned}$$

There are other ways in which these could have been done.

$$\begin{aligned} \angle B &= 180° - (90 + 46°) \\ &= 180° - 136° \\ &= 44° \end{aligned}$$

The summary of triangle ABC is as follows:

$\angle A = 46°$	$a \doteq 16$ units
$\angle B = 44°$	$b \doteq 15$ units
$\angle C = 90°$	$c = 22$ units

2. a. $\sin A = \dfrac{14}{39}$

$$\begin{aligned} \sin A &\doteq 0.358\,974\,359 \\ \angle A &\doteq 21.037\,221\,26 \\ &\doteq 21° \end{aligned}$$

$$\begin{aligned}\angle B &\doteq 180° - (90 + 21°)\\ &\doteq 180° - 111°\\ &\doteq 69°\end{aligned}$$

$$\begin{aligned}\tan B &= \frac{b}{14}\\ \tan 69° &\doteq \frac{b}{14}\\ b &\doteq 14(\tan 69°)\\ &\doteq 14 \times 2.605\,089\,065\\ &\doteq 36.471\,246\,91\\ &\doteq 36 \text{ units}\end{aligned}$$

The summary of triangle *ABC* is as follows:

$\angle A \doteq 21°$	$a = 14$ units
$\angle B \doteq 69°$	$b \doteq 36$ units
$\angle C = 90°$	$c = 39$ units

b.

$$\begin{aligned}\tan B &= \frac{b}{92}\\ \tan 56° &= \frac{b}{92}\\ b &= 92(\tan 56°)\\ &\doteq 92 \times 1.482\,560\,969\\ &\doteq 136.395\,6091\\ &\doteq 136 \text{ units}\end{aligned}$$

$$\begin{aligned}\cos B &= \frac{92}{c}\\ \cos 56° &= \frac{92}{c}\\ c &= \frac{92}{\cos 56°}\\ &\doteq \frac{92}{0.559\,192\,903}\\ &\doteq 164.522\,8318\\ &\doteq 165 \text{ units}\end{aligned}$$

$$\begin{aligned}\angle A &= 180° - (90° + 56°)\\ &= 180° - 146°\\ &= 34°\end{aligned}$$

The summary of triangle *ABC* is as follows:

$\angle A = 34°$	$a = 92$ units
$\angle B = 56°$	$b \doteq 136$ units
$\angle C = 90°$	$c \doteq 164$ units

3. To find x, use the tangent ratio.

$$\begin{aligned} \tan 37° &= \frac{1480}{x} \\ x &= \frac{1480}{\tan 37°} \\ &\doteq \frac{1480}{0.753\,554\,05} \\ &\doteq 1964.026\,336 \\ &\doteq 1964.0 \text{ units} \end{aligned}$$

To find y, use the sine ratio.

$$\begin{aligned} \tan 42° &= \frac{1480}{y} \\ y &= \frac{1480}{\tan 42°} \\ &\doteq \frac{1480}{0.900\,404\,044} \\ &\doteq 1643.706\,522 \\ &\doteq 1643.7 \text{ units} \end{aligned}$$

Activity 4

> Apply trigonometric ratios to solve word problems involving an unknown side or angle of a right triangle.

1. The angle of depression is 43°.
   The angle of elevation is represented by θ.
   Therefore, $\theta = 43°$.

$$\begin{aligned} \propto + \theta + 90° &= 180° \\ \propto + 43° + 90° &= 180° \\ \propto &= 47° \end{aligned}$$

$$\begin{aligned} \sin \theta &= \frac{x}{54} \\ \sin 43° &= \frac{x}{54} \\ x &= 54(\sin 43°) \\ &\doteq 54 \times 0.681\,998\,36 \\ &\doteq 36.827\,911\,45 \\ &\doteq 36.8 \text{ units} \end{aligned}$$

$$\sin \propto = \frac{y}{54}$$

$$\begin{aligned} \sin 47° &= \frac{y}{54} \\ y &= 54(\sin 47°) \\ &\doteq 54 \times 0.731\,135\,3701 \\ &\doteq 39.493\,099\,89 \\ &\doteq 39.5 \text{ units} \end{aligned}$$

The summary of required values is as follows:

$\propto = 47°$
$\theta = 43°$
$x \doteq 36.8$ units
$y \doteq 39.5$ units

2. The angle of depression is represented by $\propto$.
   The angle of elevation is 72°.
   Therefore, $\propto = 72°$.

$$\begin{aligned} \theta + 72° + 90° &= 180° \\ \theta &= 180° - 162° \\ &= 18° \end{aligned}$$

$$\tan \theta = \frac{y}{36}$$

$$\begin{aligned} \tan 18° &= \frac{y}{36} \\ y &= 36(\tan 18°) \\ &\doteq 36 \times 0.324\,919\,696 \\ &\doteq 11.697\,109\,06 \\ &= 11.7 \text{ units} \end{aligned}$$

$$\cos \theta = \frac{36}{x}$$

$$\begin{aligned} \cos 18° &= \frac{36}{x} \\ x &= \frac{36}{\cos 18°} \\ &\doteq \frac{36}{0.951\,056\,516} \\ &\doteq 37.852\,640\,07 \\ &\doteq 37.9 \text{ units} \end{aligned}$$

The summary of required values is as follows:

$\propto = 72°$
$\theta = 18°$
$y \doteq 11.7$ units
$x \doteq 37.9$ units

3.
$$\begin{aligned} c^2 &= a^2 + b^2 \\ c^2 &= 18^2 + 24^2 \\ c^2 &= 324 + 576 \\ c^2 &= 900 \\ \sqrt{c^2} &= \sqrt{900} \\ c &= 30.0 \text{ units} \end{aligned}$$

$$\begin{aligned} \tan B &= \frac{24}{18} \\ &\doteq 1.333\,333\,333 \\ \angle B &\doteq 53.130\,102\,35 \\ &\doteq 53^\circ \end{aligned}$$

$$\begin{aligned} \tan A &= \frac{18}{24} \\ &= 0.7500 \\ \angle A &\doteq 36.869\,897\,65 \\ &\doteq 37^\circ \end{aligned}$$

The summary of triangle ABC is as follows:

$\angle A \doteq 37^\circ$	$a = 18$ units
$\angle B \doteq 53^\circ$	$b = 24$ units
$\angle C = 90^\circ$	$c = 30$ units

4.
$$\begin{aligned} \angle A &= 180^\circ - (90^\circ + 73^\circ) \\ &= 180^\circ - 163^\circ \\ &= 17^\circ \end{aligned}$$

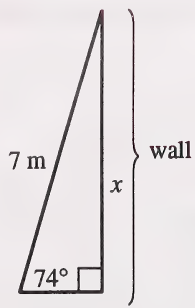
$$\begin{aligned} \sin 17^\circ &= \frac{a}{28} \\ a &= 28(\sin 17^\circ) \\ &\doteq 28 \times 0.292\,371\,704 \\ &\doteq 8.186\,407\,732 \\ &\doteq 8.2 \text{ units} \end{aligned}$$

$$\begin{aligned} \sin 73^\circ &= \frac{b}{28} \\ b &= 28(\sin 73^\circ) \\ &\doteq 28 \times 0.956\,304\,756 \\ &\doteq 26.776\,533\,17 \\ &\doteq 26.8 \text{ units} \end{aligned}$$

The summary of triangle ABC is as follows:

$\angle A = 17^\circ$	$a \doteq 8.2$ units
$\angle B = 73^\circ$	$b \doteq 26.8$ units
$\angle C = 90^\circ$	$c = 28$ units

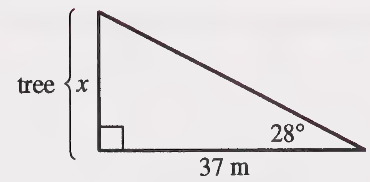
5.



$$\sin 74° = \frac{x}{7}$$
$$x = 7(\sin 74°)$$
$$\doteq 7 \times 0.961\,261\,696$$
$$\doteq 6.728\,831\,872$$
$$\doteq 6.7$$

The ladder reaches approximately 6.7 m up the wall.

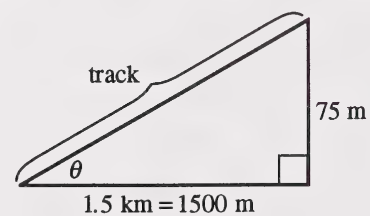
6.



$$\tan 28° = \frac{x}{37}$$
$$x = 37(\tan 28°)$$
$$\doteq 37 \times 0.531\,709\,431$$
$$\doteq 19.673\,248\,97$$
$$\doteq 19.7$$

The tree is approximately 19.7 m tall.

7.



$$\tan \theta = \frac{75}{1500}$$
$$\tan \theta = 0.0500$$
$$\theta \doteq 2.862\,405\,226$$
$$\doteq 3°$$

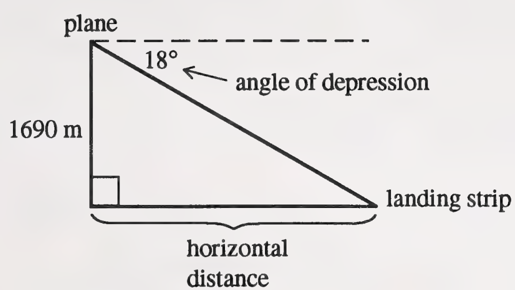
The angle of elevation of the track is approximately 3°.

8.

$$\sin 5° = \frac{x}{230}$$
$$x = 230(\sin 5°)$$
$$\doteq 230 \times 0.087\,155\,742$$
$$\doteq 20.045\,820\,83$$
$$\doteq 20.0$$

The actual rise of the road is approximately 20.0 m.

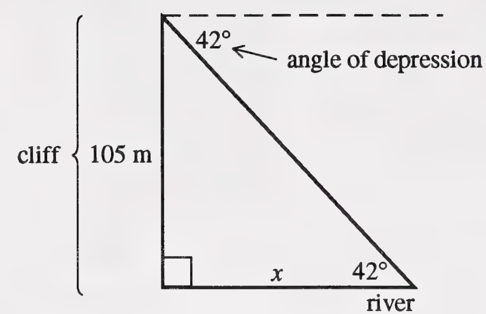
9.



$$\tan 18° = \frac{1690}{x}$$
$$x = \frac{1690}{\tan 18°}$$
$$= \frac{1690}{0.324\,919\,696}$$
$$= 5201.285\,178$$
$$= 5201$$

The plane is approximately 5201 m from the landing strip.

10.



$$\tan 42° = \frac{105}{x}$$
$$x = \frac{105}{\tan 42°}$$
$$\doteq \frac{105}{0.900\,404\,044}$$
$$\doteq 116.614\,3141$$
$$\doteq 117$$

The width of the river is approximately 117 m.

11. Find d_1 and d_2. Add d_1 and d_2 to get the distance from X to Y.

Step 1

$$\tan 53° = \frac{9}{d_1}$$

$$\begin{aligned} d_1 &= \frac{9}{\tan 53°} \\ &\doteq \frac{9}{1.327\,044\,822} \\ &\doteq 6.781\,986\,451 \text{ m} \end{aligned}$$

Step 2

$$\tan 28° = \frac{9}{d_2}$$

$$\begin{aligned} d_2 &= \frac{9}{\tan 28°} \\ &\doteq \frac{9}{0.531\,709\,431} \\ &\doteq 16.926\,538\,19 \text{ m} \end{aligned}$$

Step 3

$$\begin{aligned} XY &= d_1 + d_2 \\ &= 6.781\,986\,451 + 16.926\,538\,19 \\ &= 23.708\,524\,64 \\ &= 24 \text{ m} \quad \text{(to the nearest metre)} \end{aligned}$$

The distance between X and Y is approximately 24 m.

12. Step 1

Find the height of the tree.

$$\tan 35° = \frac{x}{52}$$

$$\begin{aligned} x &= 52(\tan 35°) \\ &\doteq 52 \times 0.700\,207\,538 \\ &\doteq 36.410\,791\,99 \text{ m} \end{aligned}$$

Step 2

Find distance y.

$$\tan 43° = \frac{x}{y}$$

$$\begin{aligned} y &\doteq \frac{36.410\,791\,99}{\tan 43°} \\ &\doteq \frac{36.410\,791\,99}{0.932\,515\,086} \\ &\doteq 39.045\,794\,04 \\ &\doteq 39 \text{ m} \end{aligned}$$

Distance y is 39 m.

Step 3

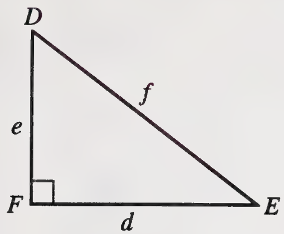
Add distance y to 52 m.

$52 \text{ m} + 39 \text{ m} \doteq 91 \text{ m}$

The distance between Jackie and Saundra is about 91 m.

Extra Help

1.



$\sin D = \frac{d}{f}$ $\sin E = \frac{e}{f}$

$\cos D = \frac{e}{f}$ $\cos E = \frac{d}{f}$

$\tan D = \frac{d}{e}$ $\tan E = \frac{e}{d}$

2. The fraction form of $\angle A$ is as follows:

$\sin A = \frac{7}{11}$ $\cos A = \frac{5}{11}$ $\tan A = \frac{7}{5}$

The decimal form of $\angle A$ is as follows:

$\sin A = 0.636\,36$ $\cos A = 0.454\,54$ $\tan A = 1.400\,00$

$= 0.6364$ $= 0.4545$ $= 1.4000$

The fraction form of $\angle B$ is as follows:

$\sin B = \frac{5}{11}$ $\cos B = \frac{7}{11}$ $\tan B = \frac{5}{7}$

The decimal form of $\angle B$ is as follows:

$\sin B = 0.454\,54$ $\cos B = 0.636\,36$ $\tan B = 0.714\,28$

$= 0.4545$ $= 0.6364$ $= 0.7143$

3. a.

Enter	Display
0.7042	0.7042
INV	0.7042
cos	45.23505055

Therefore, $\angle A \doteq 45°$.

b.

Enter	Display
0.2009	0.2009
INV	0.2009
sin	11.58959351

Therefore, $\angle A \doteq 12°$.

c.

Enter	Display
1.7634	1.7634
INV	1.7634
tan	60.44302132

Therefore, $\angle A \doteq 60°$.

d.

Enter	Display
0.5036	0.5036
INV	0.5036
tan	26.72982529

Therefore, $\angle B \doteq 27°$.

e.

Enter	Display
0.8961	0.8961
INV	0.8961
sin	63.65007274

Therefore, $\angle B \doteq 64°$.

f.

Enter	Display
0.0337	0.0337
INV	0.0337
cos	88.06876657

Therefore, $\angle B \doteq 88°$.

4. $$\begin{aligned} \angle A &= 180° - (90° + 51°) \\ &= 180° - 141° \\ &= 39° \end{aligned}$$

$$\begin{aligned} \tan\ B &= \frac{33}{a} \\ \tan\ 51° &= \frac{33}{a} \\ a &= \frac{33}{\tan\ 51°} \\ &\doteq \frac{33}{1.234\,897\,157} \\ &\doteq 26.722\,8731 \\ &\doteq 27 \text{ units} \end{aligned}$$

$$\sin B = \frac{33}{c}$$

$$\sin 51° = \frac{33}{c}$$

$$c = \frac{33}{\sin 51°}$$

$$\doteq \frac{33}{0.777\,145\,961}$$

$$\doteq 42.463\,065\,67$$

$$\doteq 42 \text{ units}$$

The summary of triangle ABC is as follows:

$\angle A = 39°$	$a \doteq 27$ units
$\angle B = 51°$	$b = 33$ units
$\angle C = 90°$	$c \doteq 42$ units

5. $\tan 36° = \frac{x}{420}$

$$x = 420(\tan 36°)$$

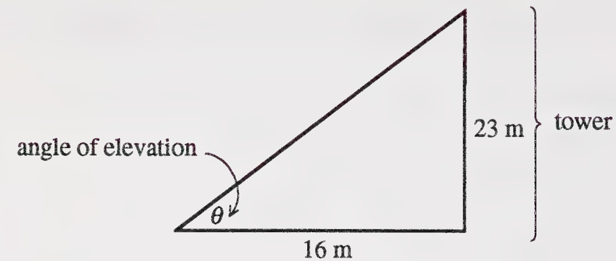
$$\doteq 420 \times 0.726\,542\,528$$

$$\doteq 305.147\,8618$$

$$\doteq 305 \text{ m}$$

The helicopter is approximately 305 m above the landing pad.

6.



$$\tan \theta = \frac{23}{16}$$

$$\tan \theta = 1.4375$$

$$\theta \doteq 55.175\,510\,84$$

$$\doteq 55°$$

The angle of elevation is approximately 55°.

Extensions

1. Find the base of the larger triangle first. The length is also the hypotenuse of the smaller triangle. This side is labelled x.

$$\begin{aligned} c^2 &= a^2 + b^2 \\ (20.8)^2 &= x^2 + (19.2)^2 \\ 432.64 &= x^2 + 368.64 \\ x^2 &= 432.64 - 368.64 \\ x^2 &= 64 \\ \sqrt{x^2} &= \sqrt{64} \\ x &= 8 \end{aligned}$$

$$\begin{aligned} \text{Area} &= \frac{1}{2} \times \text{base} \times \text{height} \\ &= \frac{1}{2}(8)(19.2) \\ &= 76.8 \text{ square units} \end{aligned}$$

The base of the larger triangle is 8 units, and the perpendicular height is 19.2 units.

Now find y in the smaller triangle.

$$\begin{aligned} c^2 &= a^2 + b^2 \\ 8^2 &= 6.4^2 + y^2 \\ 64 &= 40.96 + y^2 \\ y^2 &= 64 - 40.96 \\ y^2 &= 23.04 \\ \sqrt{y^2} &= \sqrt{23.04} \\ y &= 4.8 \end{aligned}$$

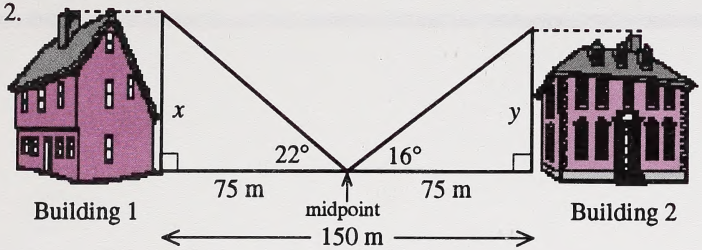
$$\begin{aligned} \text{Area} &= \frac{1}{2} \times \text{base} \times \text{height} \\ &= \frac{1}{2}(4.8)(6.4) \\ &= 15.36 \text{ square units} \end{aligned}$$

The base of the smaller triangle is 6.4 units, and the perpendicular height is 4.8 units.

Now find the combined area of the two triangles.

$$\begin{aligned} \text{total area} &= \text{area of } \Delta 1 + \text{area of } \Delta 2 \\ &= 76.8 + 15.36 \\ &= 92.16 \\ &\doteq 92.2 \text{ square units} \end{aligned}$$

2.



Find x (the height of Building 1).

$$\tan 22° = \frac{x}{75}$$

$$x = 75(\tan 22°)$$

$$\doteq 75(0.404\,026\,225)$$

$$\doteq 30.301\,966\,94$$

$$\doteq 30.3 \text{ m}$$

Find y (the height of Building 2).

$$\tan 16° = \frac{y}{75}$$

$$y = 75(\tan 16°)$$

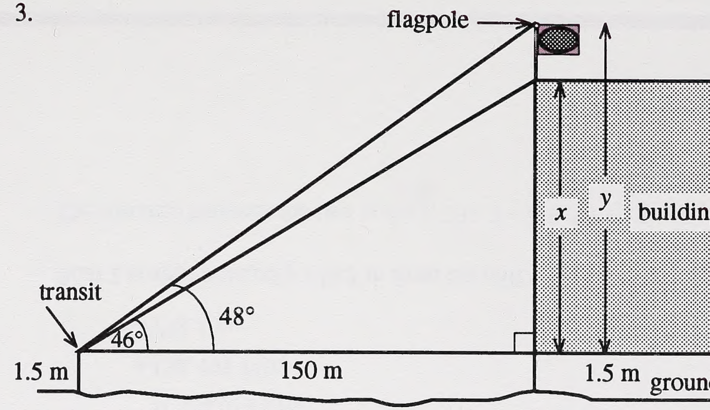
$$\doteq 75(0.286\,745\,385)$$

$$\doteq 21.505\,903\,93$$

$$\doteq 21.5 \text{ m}$$

The taller building is $30.3 - 21.5 \doteq 8.8$ m taller then the shorter building.

3.



Let the height of the building be x, and let y be the height of the building plus the flagpole.

Height of building (x) is as follows:

$$\tan 46° = \frac{x}{150}$$

$$x = 150(\tan 46°)$$

$$\doteq 150(1.035\,530\,314)$$

$$\doteq 155.329\,5471$$

$$\doteq 155.3 \text{ m}$$

The height of the building is $155.3 + 1.5 \doteq 156.8$ m.

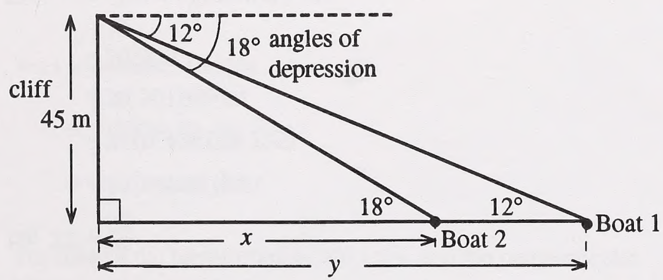
Height of the building and the flapole (y) is as follows:

$$\tan 48° = \frac{y}{150}$$
$$y = 150(\tan 48°)$$
$$\doteq 150(1.110\,612\,515)$$
$$\doteq 166.591\,8772$$
$$\doteq 166.6 \text{ m}$$

The height of the building plus the flagpole is $166.6 + 1.5 \doteq 168.1$ m.

The height of the flagpole alone is $168.1 - 156.8 \doteq 11.3$ m.

4.



Find y (Boat 1).

$$\tan 12° = \frac{45}{y}$$
$$y = \frac{45}{\tan 12°}$$
$$\doteq \frac{45}{0.212\,556\,561}$$
$$\doteq 211.708\,3549$$
$$\doteq 211.7 \text{ m}$$

Boat 1 is approximately 211.7 m from the cliff.

Find x (Boat 2).

$$\tan 18° = \frac{45}{x}$$
$$x = \frac{45}{\tan 18°}$$
$$\doteq \frac{45}{0.324\,919\,696}$$
$$\doteq 138.495\,7592$$
$$\doteq 138.5 \text{ m}$$

Boat 2 is approximately 138.5 m from the cliff.

The distance between the two boats is $211.7 - 138.5 \doteq 73.2$ m.

L.R.D.C.
Producer

Mathematics 10

Student Module
Unit 7

1992